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Sustainable Electrified Transportation Center
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Mutual Resistance

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July 23, 2020

Revised March 8, 2021

Overview

- This presentation introduces a circuit simulation model that includes both inductive coupling and mutual resistance effects
- This circuit model is more general than typical equivalent circuits
  - Models coupling effects such as leakage inductances, cross regulation and ripple steering
  - Models skin and proximity effect losses in windings and shields
  - Shows how winding losses vary during a switching cycle

Dowell's Method (what the model is intended to replace)

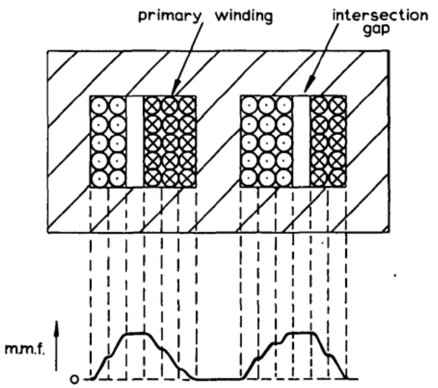


Fig. 2
A typical transformer, with associated m.m.f. diagram
○ ⊗ Indicates current directions

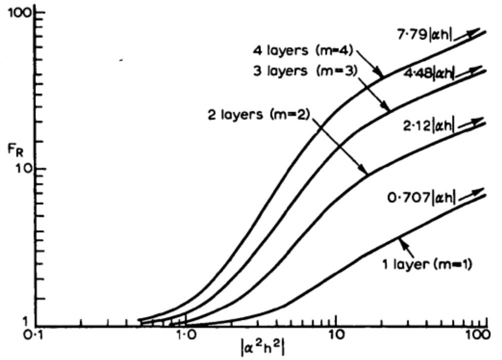


Fig. 8
F_R as a function of $|\alpha^2h^2|$ for winding portions with integral numbers of layers

- Used to calculate the ac resistance of transformer windings [1]
- Assumes infinite magnetizing inductance (equal and opposite amp-turns)
  - not intended for low permeability or gapped cores (amp-turns unequal and possible fringing loss)
- Assumes one independent current variable (currents scaled by turns ratios)
  - Interleaved windings are allowed if they are connected in series
  - Multiple outputs with independent load currents not allowed
  - Windings connected in parallel not allowed because the current sharing ratio is unknown

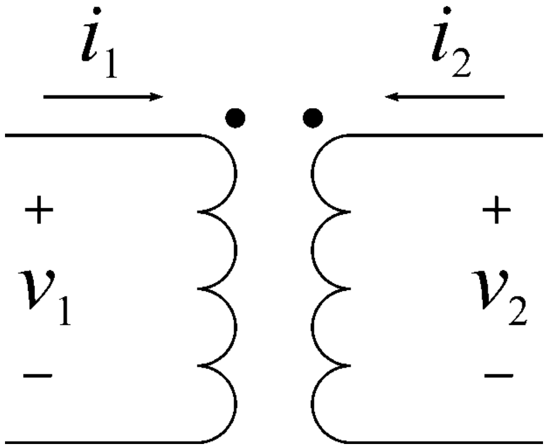
Advantages of the new Mutual Impedance Circuit Model

- Provides a general model for both time and frequency domains
- Models coupling effects such as leakage inductances, cross regulation, and ripple steering
- Models both skin and proximity effects for any winding configuration
- Models losses due to fringing fields from gapped cores
- Works with transformers supplying multiple loads
- Models losses and current sharing when windings are connected in parallel
- Shows how the shape of the current waveforms affect losses
- The model can be derived from:
  - Finite Element Analysis
  - Measured impedance data (accuracy is improved if capacitive effects are subtracted)
  - Geometry based equations (More research is needed to further develop this)

Self and Mutual Impedance Modeling

- Coupled windings each have their own impedance called a self-impedance that has resistive and inductive parts
- Each pair of windings has a mutual impedance that also has resistive and inductive parts
  - Mutual inductances
  - The real part of the mutual impedance is called the mutual resistance [5-10]
- Mutual inductances are modeled with coupled inductors and coupling coefficients
- Mutual resistances are modeled with auxiliary windings loosely coupled to the physical windings
- The skin and proximity effects are modeled by resistors in parallel with the auxiliary windings

Mutual Impedance Equations



Z_{11} = Winding 1 Self Impedance

Z_{22} = Winding 2 Self Impedance

$Z_{12} = Z_{21}$ = Mutual Impedance

$R_{12} = R_{21}$ = Mutual Resistance [5-10]

$L_{12} = L_{21}$ = Mutual Inductance

$$v_1 = Z_{11} i_1 + Z_{12} i_2$$
$$= (R_{11} + j\omega L_{11}) i_1 + (R_{12} + j\omega L_{12}) i_2$$

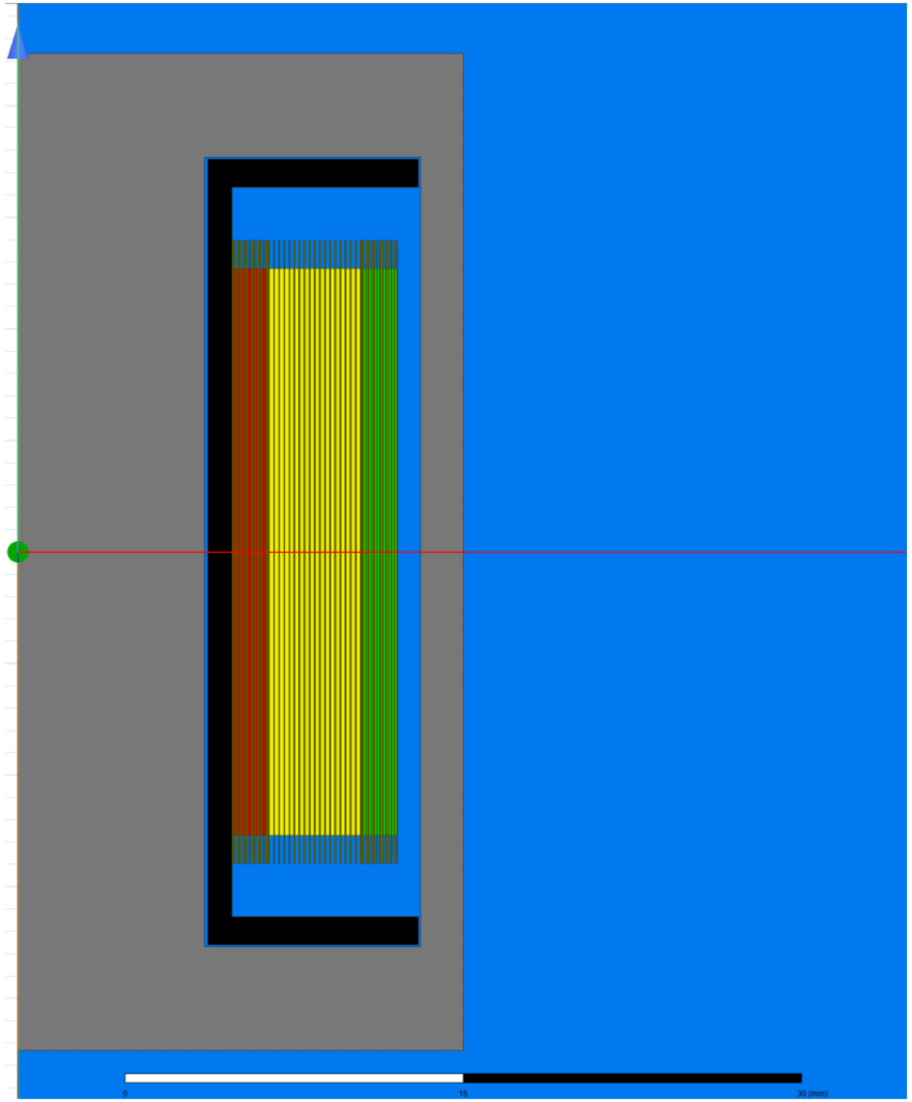
$$v_2 = Z_{21} i_1 + Z_{22} i_2$$
$$= (R_{21} + j\omega L_{21}) i_1 + (R_{22} + j\omega L_{22}) i_2$$

Mutual Impedance for N Windings

$$\begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1N} \\ Z_{21} & Z_{22} & \cdots & Z_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{N1} & Z_{N2} & \cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_N \end{bmatrix}$$

- Finite element analysis (FEA) software can compute a matrix of the resistive and inductive parts of the impedance matrix using an eddy current solver
- Each current-carrying winding needs to be added to the matrix calculation
- Losses from non-current carrying conductors such as shields are automatically included in the losses of the windings included in the matrix
- Losses due to core gaps are automatically included in the model
- A lossless core model is best for FEA modeling of winding losses

Example Transformer



- Maxwell 2D radial model for a core with a round center leg
- Impedance matrix data for several frequencies was exported as a text file and then imported into Excel and then Mathcad

Core: **ETD49-25-16-3C95**

The core permeability is reduced to 428 to simulate having a 3-mil gap between the core halves.

Winding 1: 12 Turns 3 mil foil

Winding 2: 18 Turns 7 mil foil

Winding 3: 12 Turns 3 mil foil

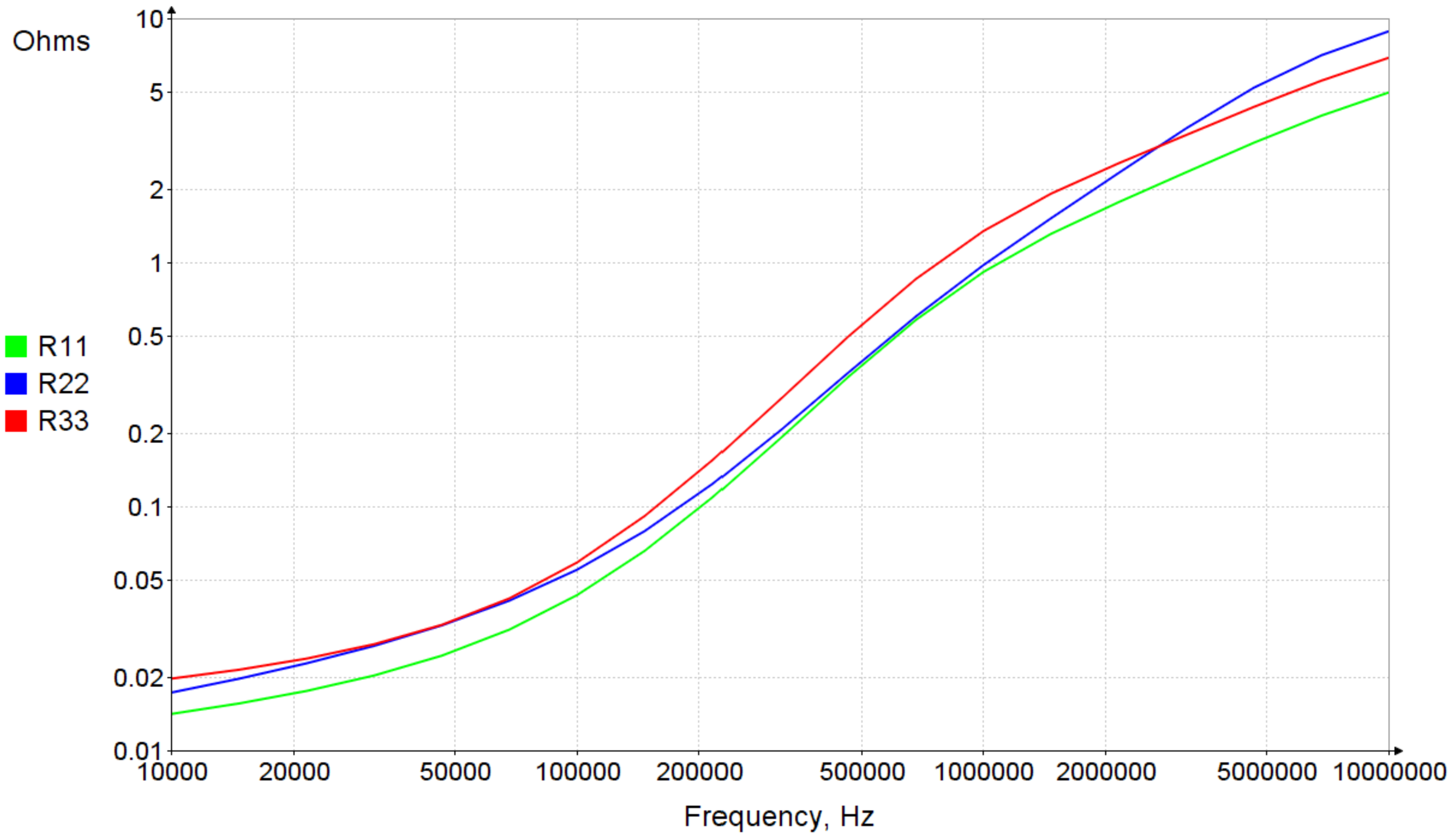
Layer insulation: 2 mil Nomex

Inter-Winding insulation: 3 layers 2 mil Nomex

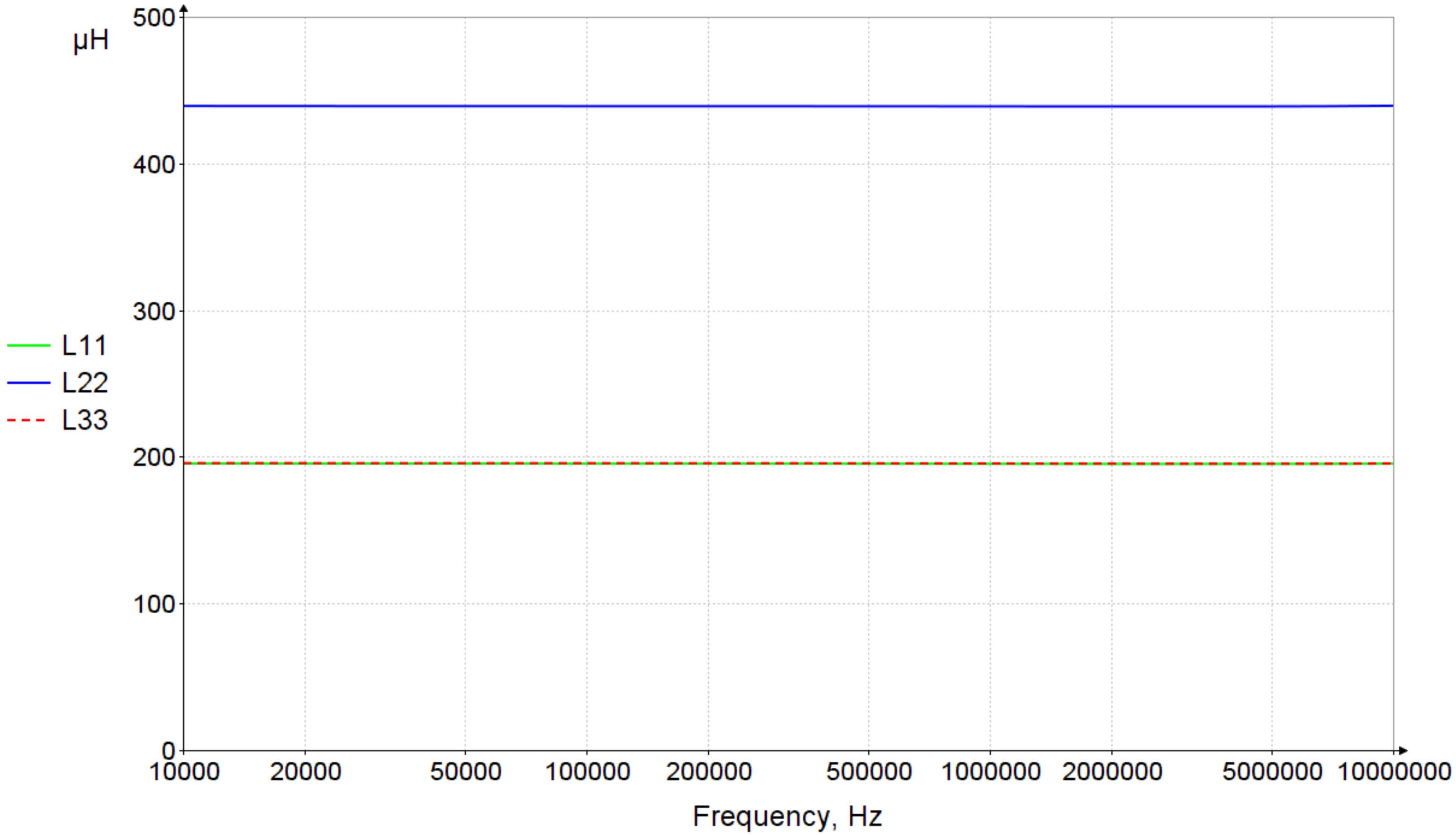
```
10000Hz
    R,L
            Winding_1   Winding_2   Winding_3
        Winding_1   0.014102, 1.9539E+05    0.0062315, 2.9223E+05   0.0034226, 1.9418E+05
        Winding_2   0.0062315, 2.9223E+05   0.017256, 4.3913E+05    0.0064318, 2.9226E+05
        Winding_3   0.0034226, 1.9418E+05   0.0064318, 2.9226E+05   0.019679, 1.9567E+05

14677.9926762207Hz
    R,L
            Winding_1   Winding_2   Winding_3
        Winding_1   0.015564, 1.9538E+05    0.0076353, 2.9221E+05   0.0040162, 1.9417E+05
        Winding_2   0.0076353, 2.9221E+05   0.019661, 4.391E+05 0.0079112, 2.9224E+05
        Winding_3   0.0040162, 1.9417E+05   0.0079112, 2.9224E+05   0.0214, 1.9565E+05
```

Self Resistances

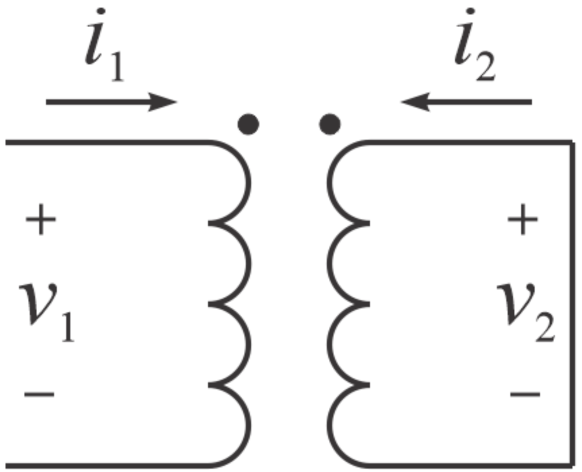


Self Inductances



Leakage Impedance

Z_{leak_mn}



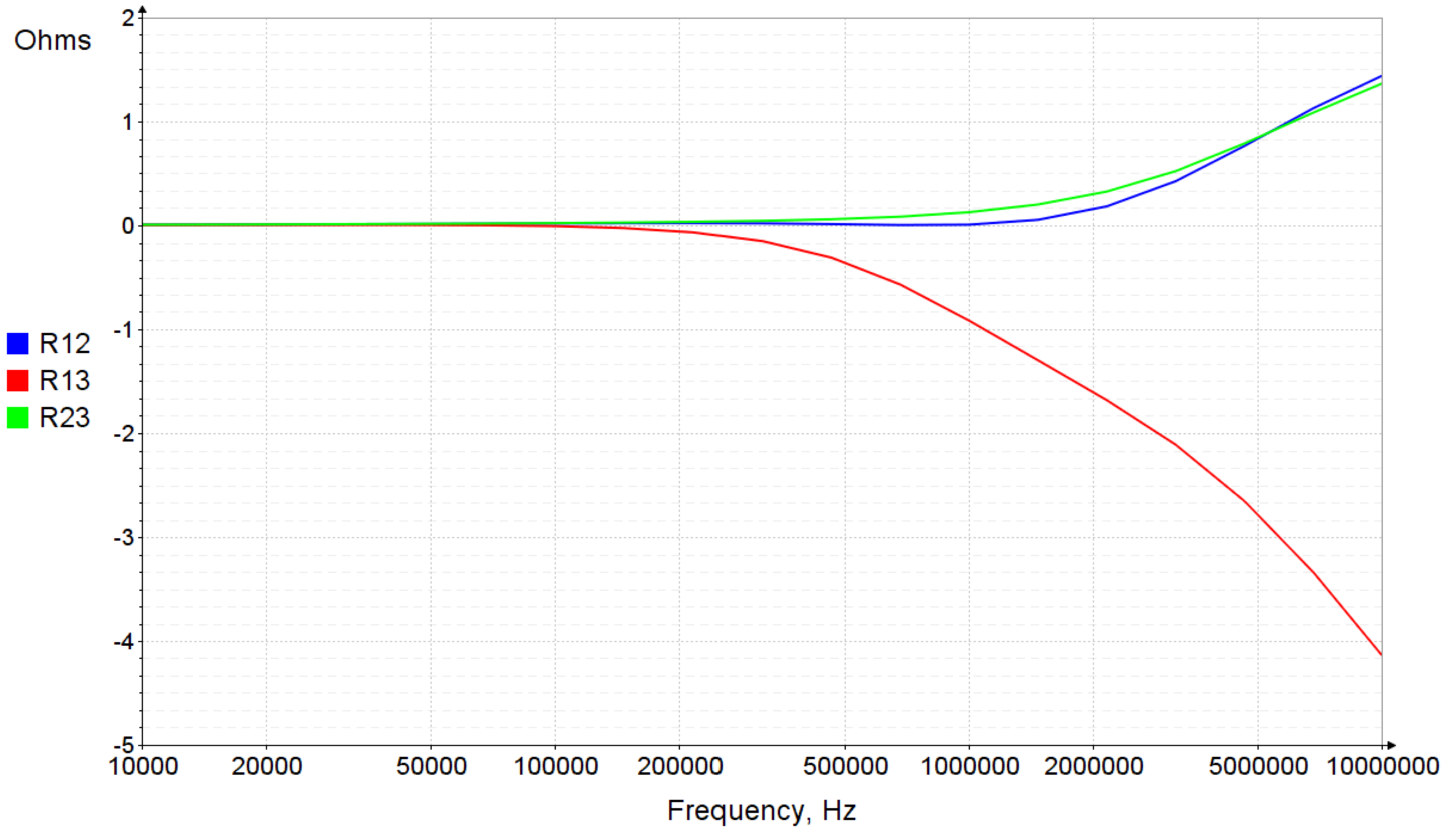
Winding m

Winding n shorted

$$Z_{leak\_mn} = Z_{mm} - \frac{{Z_{mn}}^2}{Z_{nn}}$$

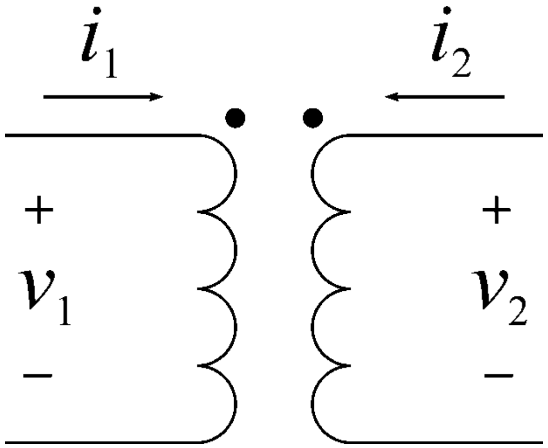
- Leakage impedances can be derived from impedance matrix data produced by FEA software by using the above formula

Mutual Resistances



- Mutual resistances can be positive or negative, and can change sign at different frequencies depending on the interactions of the magnetic fields and eddy currents

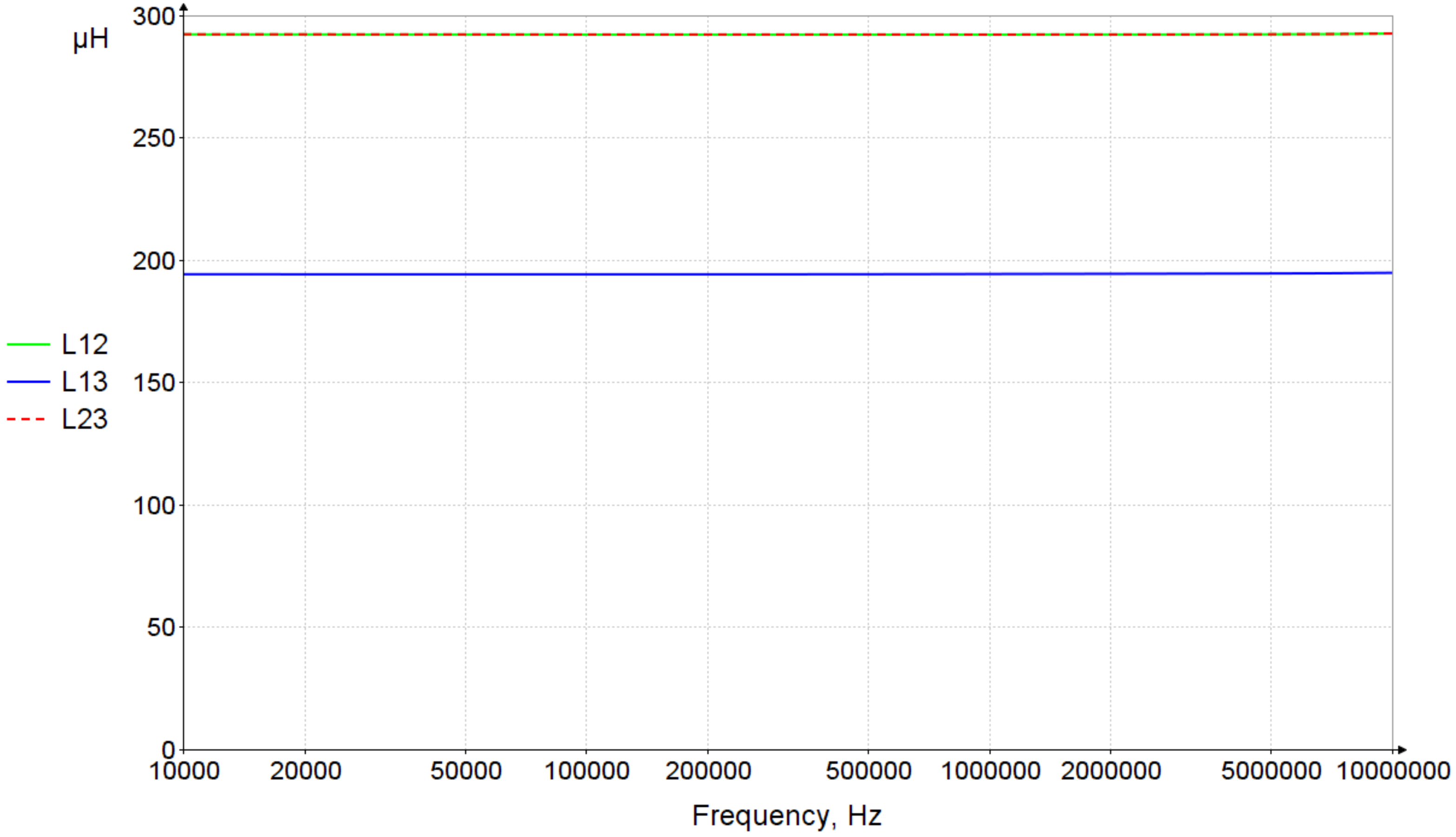
Power Dissipation



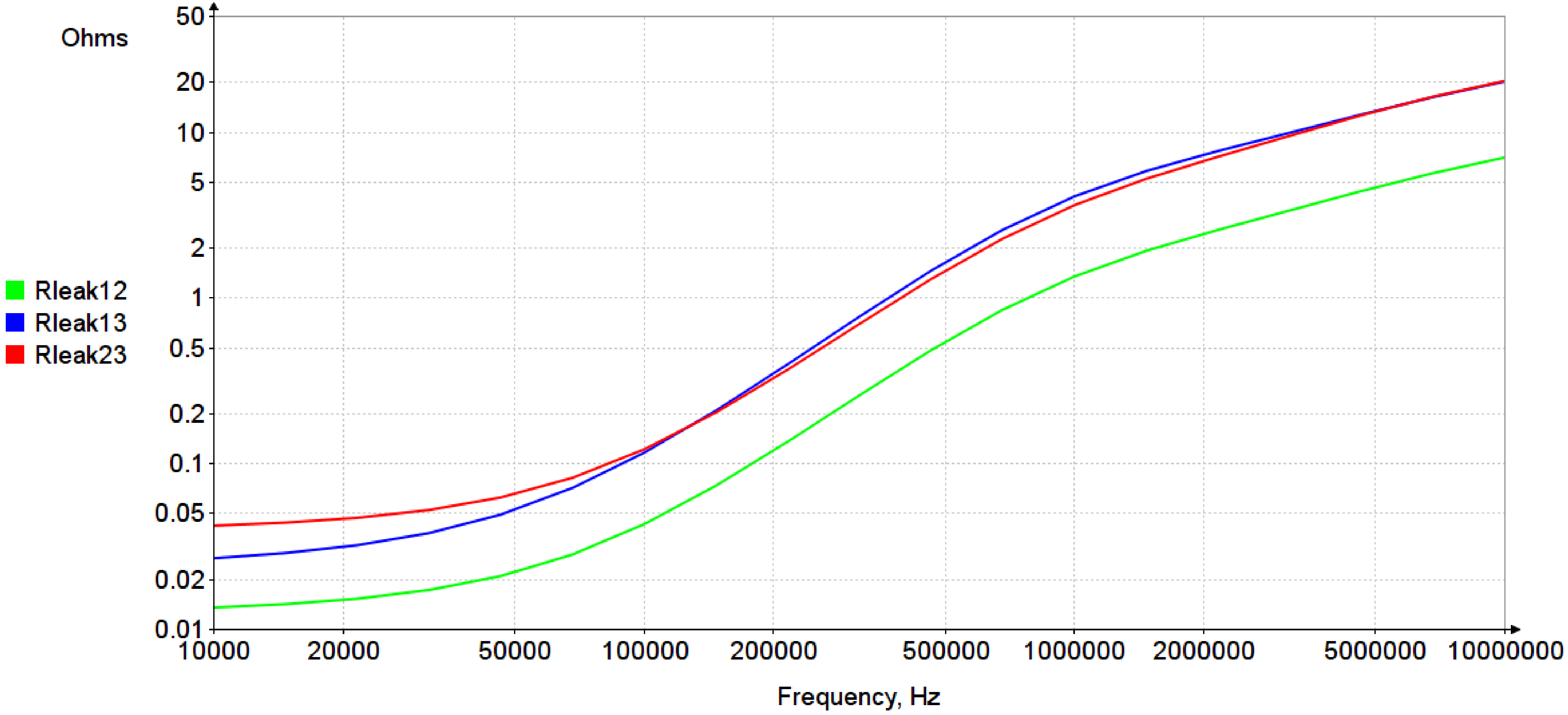
$$P = \frac{1}{2}R_{11}|i_1|^2 + \frac{1}{2}R_{22}|i_2|^2 + R_{12}\mathrm{Re}(i_1 i_2^*) \quad [8]$$

- For typical transformers, i_1 and i_2, with the conventional polarities shown above, have opposite signs and minimal phase shift, so the last term is negative when the mutual resistance is positive, and this reduces the losses
- In contrast, the losses are increased when R_{12} is negative (usually at high frequencies)
- The total power is always positive

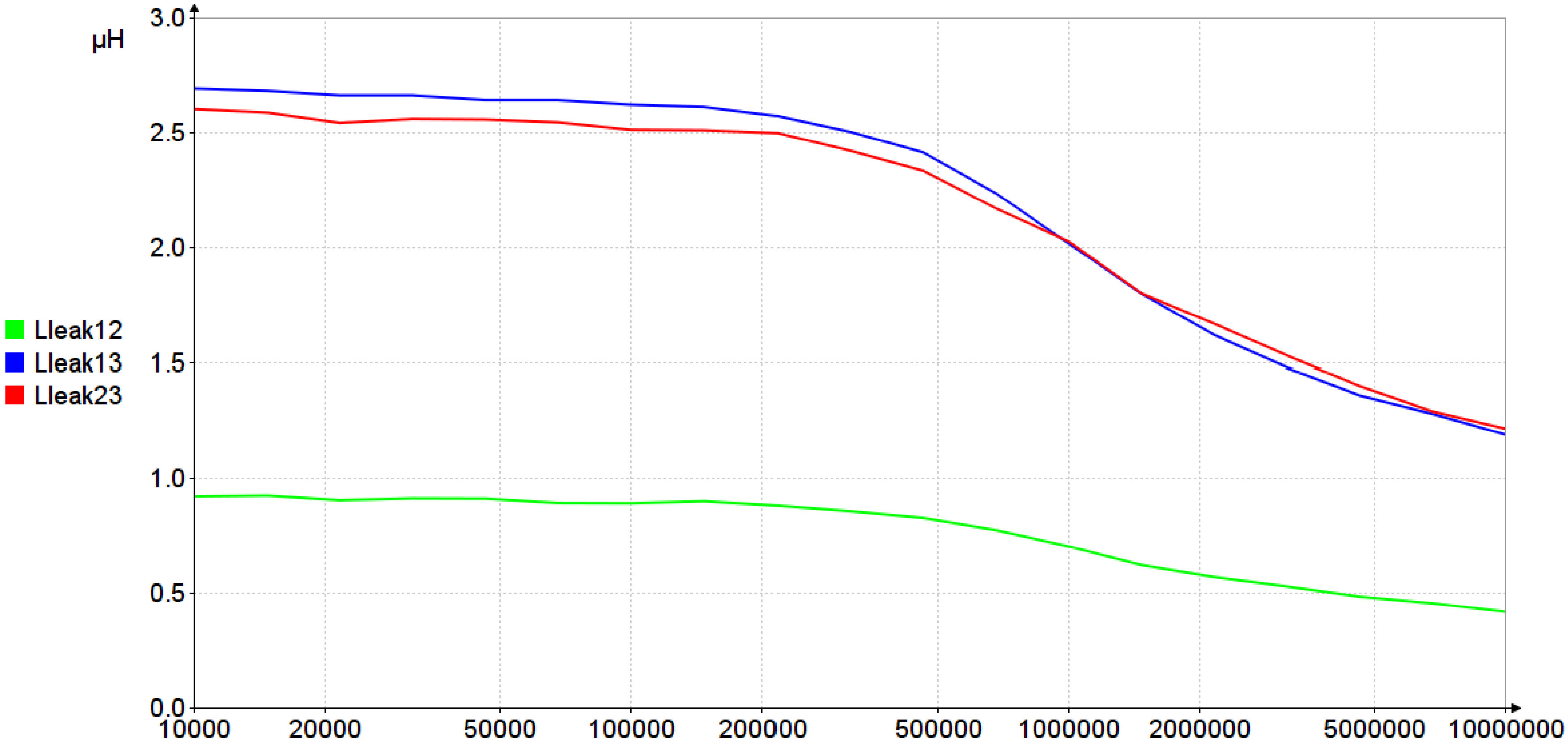
Mutual Inductances



Leakage Resistances



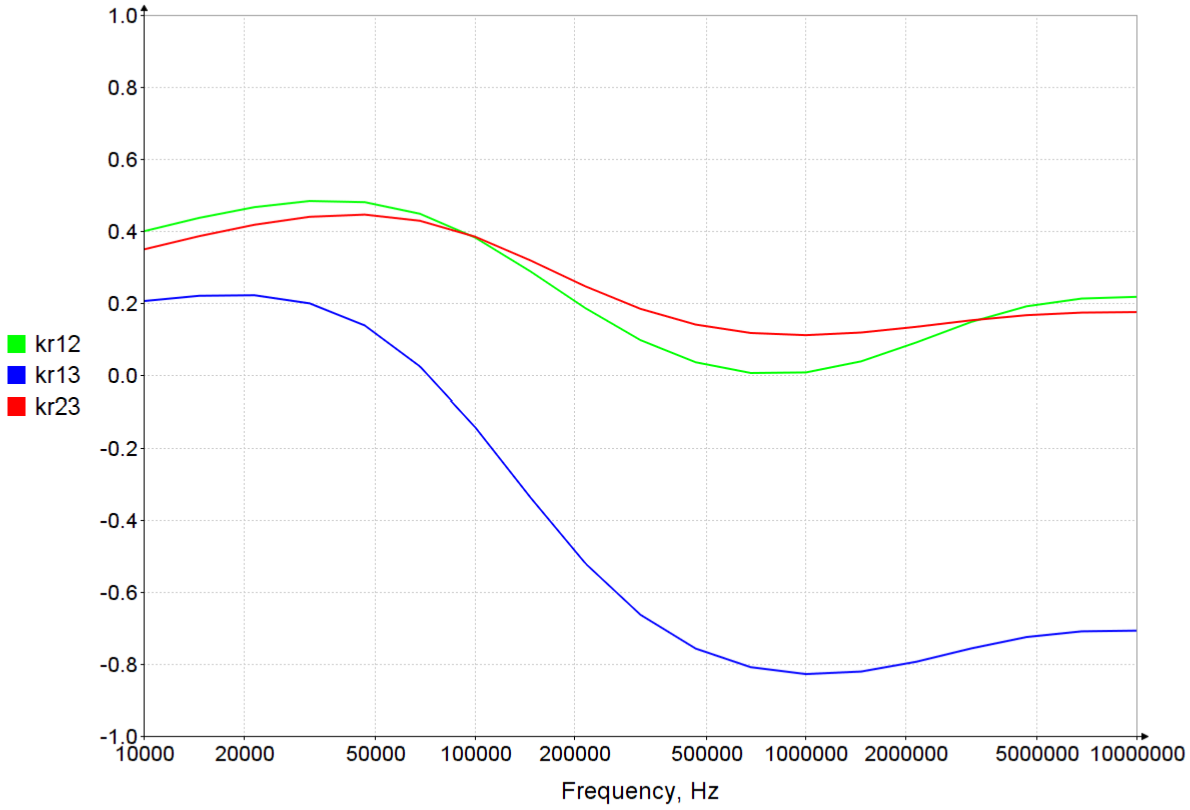
Leakage Inductances



Mutual Resistance Coupling

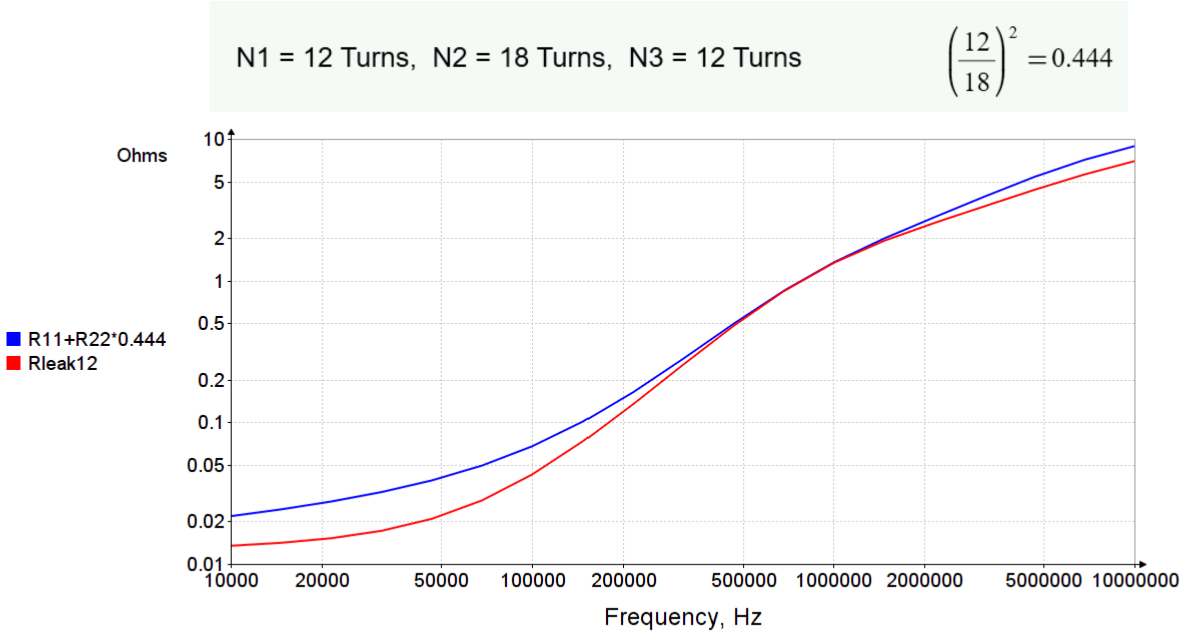
$$k_{mn}^{R} = \frac{R_{mn}}{\sqrt{R_{mm}R_{nn}}}$$

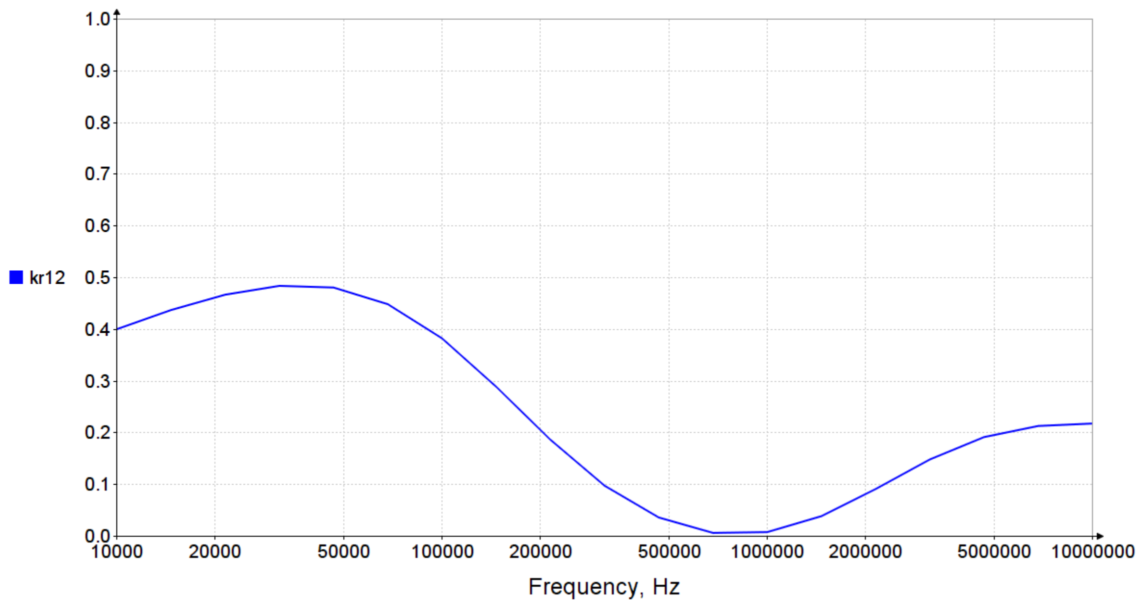
$$-1 < k_{mn}^{R} < 1$$



- A coupling coefficient for mutual resistance k^R can be defined as shown above [10]
- The coupling coefficient is negative when the mutual resistance is negative
- The inequality ensures that the total dissipated power is always positive for two windings
- More restrictive criteria are needed to ensure system stability for more than two windings

Effects of Mutual Resistance on Leakage Resistance

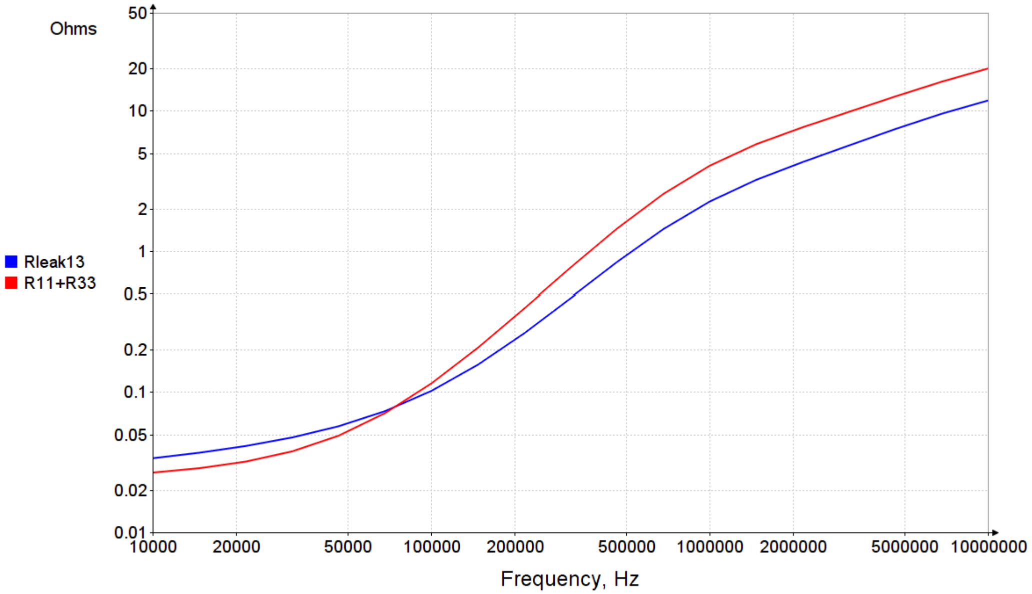


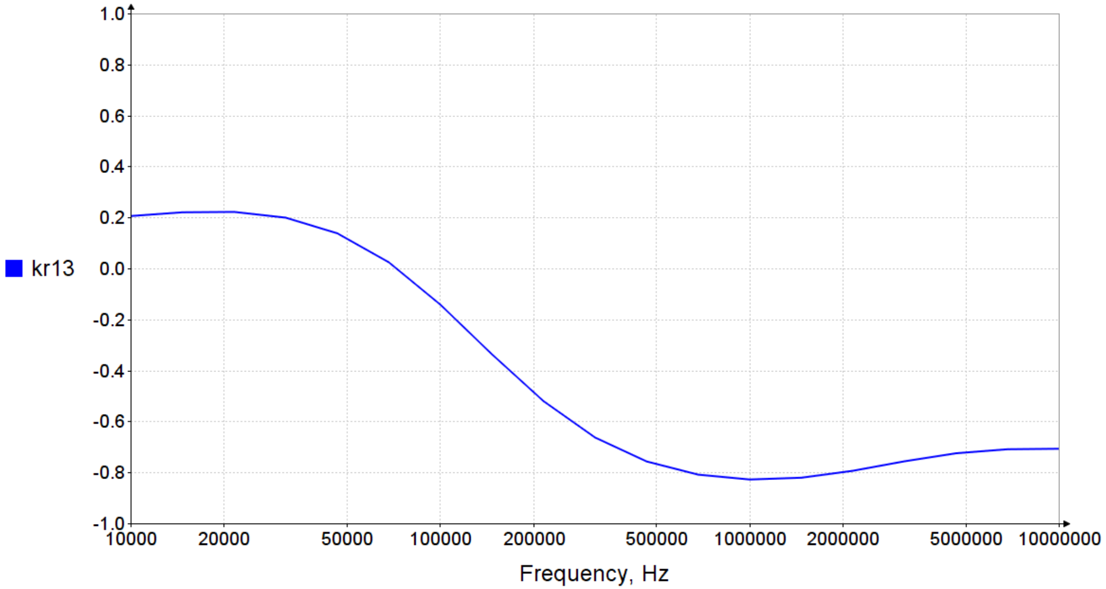


- When the mutual resistance coupling coefficient is positive, the leakage resistance (red) is less than the sum of the primary self-resistance and the reflected secondary self-resistance (blue)
- The reduction in leakage resistance is diminished when the mutual resistance coupling coefficient heads toward zero

Effects of Mutual Resistance on Leakage Resistance

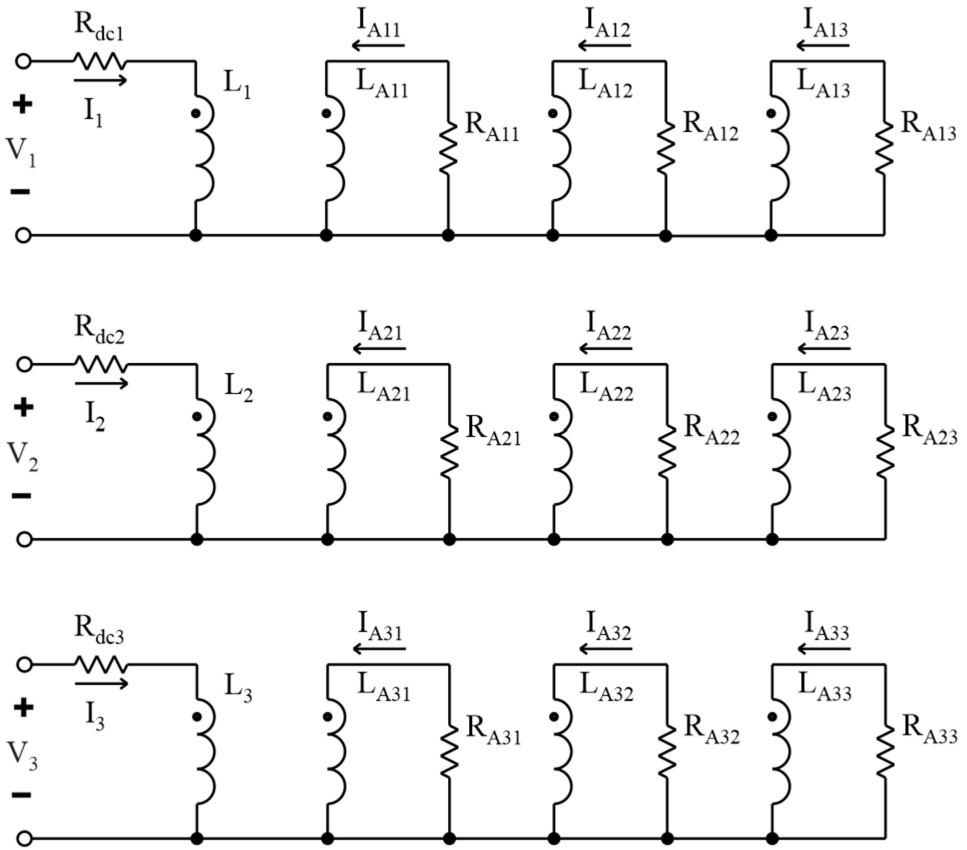
N1 = 12 Turns, N2 = 18 Turns, N3 = 12 Turns $\left(\frac{12}{12}\right)^2 = 1$





- When the mutual resistance coupling coefficient is negative, the leakage resistance (red) is more than the sum of the primary self-resistance and the and the reflected secondary self-resistance (blue)
- When the mutual resistance coupling coefficient is negative, then the leakage resistance reduction doesn’t occur and instead the losses increase

An Equivalent RL Circuit for a Three-Winding Transformer




```
.param Lb1=0.00019592
+      Lb2=0.00044032
+      Lb3=0.00019623
.param Rb1=0.0089431
+      Rb2=0.0073656
+      Rb3=0.013973
.param RA11=1225.25771025044
+      RA12=1158.40036754399
+      RA13=181.325764013633
+      RA21=19100.9241533973
+      RA22=16047.4362349169
+      RA23=9141.01291010845
+      RA31=59.0271237686648
+      RA32=24.7459906542664
+      RA33=9.9999997343195
KA1 Lb1 LA11 -0.0087469732226671
KA2 Lb1 LA12 0.0335756534803779
KA3 Lb1 LA13 0.00651435979331905
KA4 Lb1 LA21 0.0205912922181521
KA5 Lb1 LA22 0.00987088632524521
KA6 Lb1 LA23 0.0117694769413289
KA7 Lb1 LA31 0.0115981263243377
KA8 Lb1 LA32 -0.00881441787398592
KA9 Lb1 LA33 0.0247100977857254
KA10 Lb2 LA11 0.0209212861034966
KA11 Lb2 LA12 0.00189083029426342
KA12 Lb2 LA13 0.0104505361349864
KA13 Lb2 LA21 -0.00341436251900779
KA14 Lb2 LA22 0.024562007353195
KA15 Lb2 LA23 0.00455882073697005
KA16 Lb2 LA31 0.00551547113846126
KA17 Lb2 LA32 0.000453390289046192
KA18 Lb2 LA33 0.0258705678044811
KA19 Lb3 LA11 0.00543963848648762
KA20 Lb3 LA12 -0.0423721547330175
KA21 Lb3 LA13 0.00739971309374588
KA22 Lb3 LA21 -0.026490587740296
KA23 Lb3 LA22 0.000803727518113103
KA24 Lb3 LA23 0.0129937292106632
KA25 Lb3 LA31 0.0106998580980133
KA26 Lb3 LA32 0.0120961005269526
KA27 Lb3 LA33 0.02503605965727
Kb1 Lb1 Lb2 0.997571488659212
Kb2 Lb1 Lb3 0.992834684818819
Kb3 Lb2 Lb3 0.99695330557125
```

- Model based methods described in [2, 3]
- The physical windings are represented by L1,L2 and L3
- Each physical winding is accompanied by two auxiliary windings shunted by resistors that model the ac losses
- Each physical winding is coupled to each of the auxiliary windings
- The auxiliary windings are not coupled to each other in this model, but they are in the model described in [2]
- Some of the couplings are negative
- The auxiliary windings have the same inductance as their associated physical winding
- The parameter values were determined by a solver in Mathcad that attempts to match the performance of the model to the impedance matrix data imported from Maxwell
- The solver details are not discussed in this presentation
- The solver used in this presentation is still in the process of being optimized
- The frequency range of the model can be extended by using more than two auxiliary windings for each physical winding

Stability Check

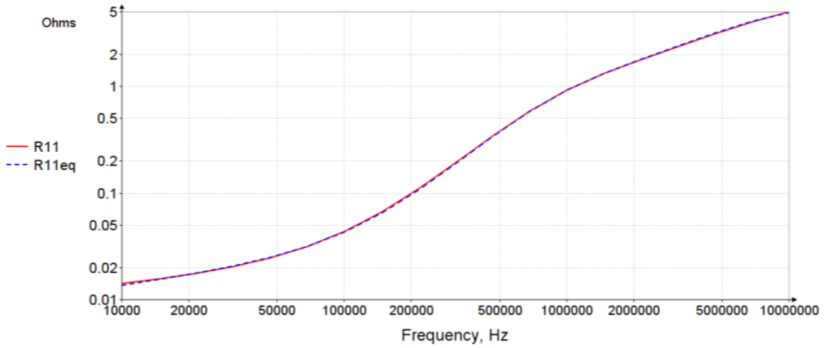
$$K_{sys} := \begin{pmatrix} 1 & Kb_{1,2} & Kb_{1,3} & k_{A_1} & k_{A_2} & k_{A_3} & k_{A_4} & k_{A_5} & k_{A_6} \\ Kb_{2,1} & 1 & Kb_{2,3} & k_{A_7} & k_{A_8} & k_{A_9} & k_{A_{10}} & k_{A_{11}} & k_{A_{12}} \\ Kb_{3,1} & Kb_{3,2} & 1 & k_{A_{13}} & k_{A_{14}} & k_{A_{15}} & k_{A_{16}} & k_{A_{17}} & k_{A_{18}} \\ k_{A_1} & k_{A_7} & k_{A_{13}} & 1 & 0 & 0 & 0 & 0 & 0 \\ k_{A_2} & k_{A_8} & k_{A_{14}} & 0 & 1 & 0 & 0 & 0 & 0 \\ k_{A_3} & k_{A_9} & k_{A_{15}} & 0 & 0 & 1 & 0 & 0 & 0 \\ k_{A_4} & k_{A_{10}} & k_{A_{16}} & 0 & 0 & 0 & 1 & 0 & 0 \\ k_{A_5} & k_{A_{11}} & k_{A_{17}} & 0 & 0 & 0 & 0 & 1 & 0 \\ k_{A_6} & k_{A_{12}} & k_{A_{18}} & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

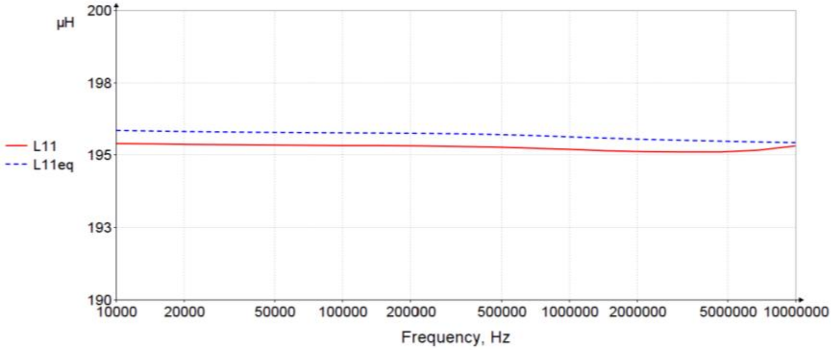
If negative eigenvalues values are present then the reliazability criterion is violated

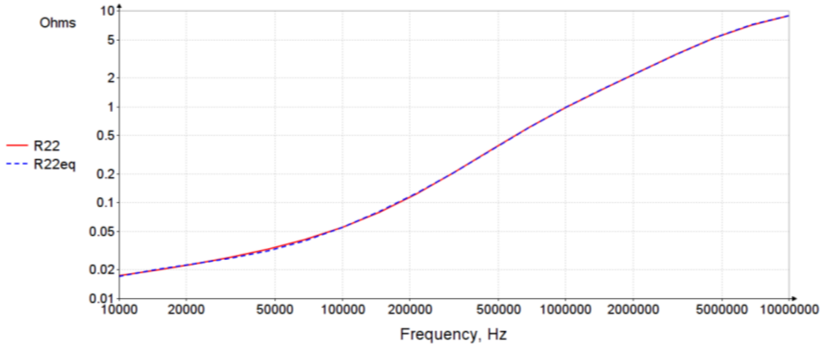
$$\text{eigenvals}\left(K_{sys}\right) = \begin{bmatrix} 2.993 \\ 1.004 \\ 1.001 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 0.998 \\ 0.003 \\ 5.642 \cdot 10^{-4} \end{bmatrix}$$

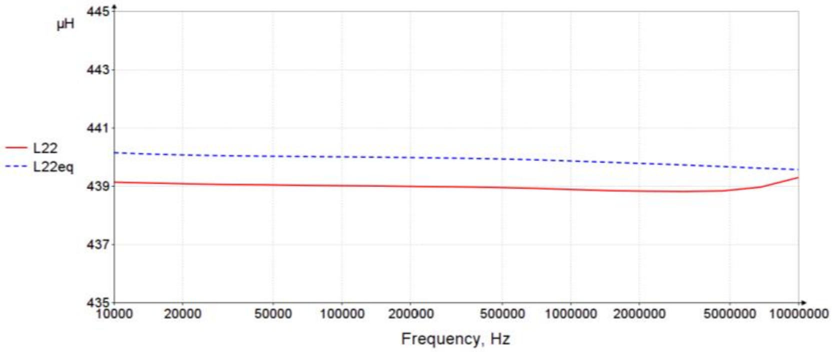
- A matrix of all the coupling coefficients used in the model can be used to check the stability of the model [4]
- The solver checks for stability and rejects unstable solutions
- The model is reduced-order because not all the couplings are included (the model in [2] includes all couplings)

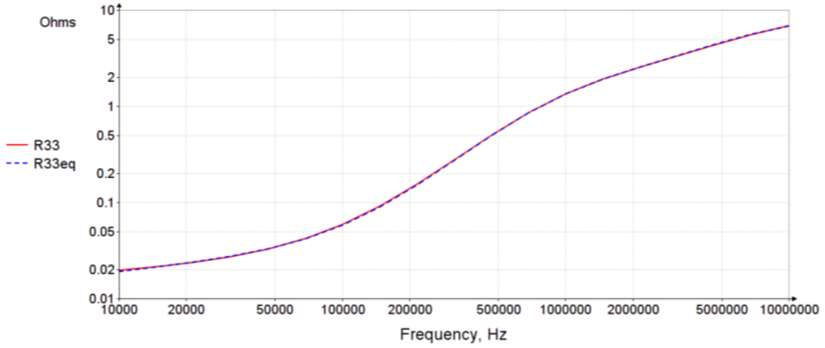
Self Resistances and Inductances

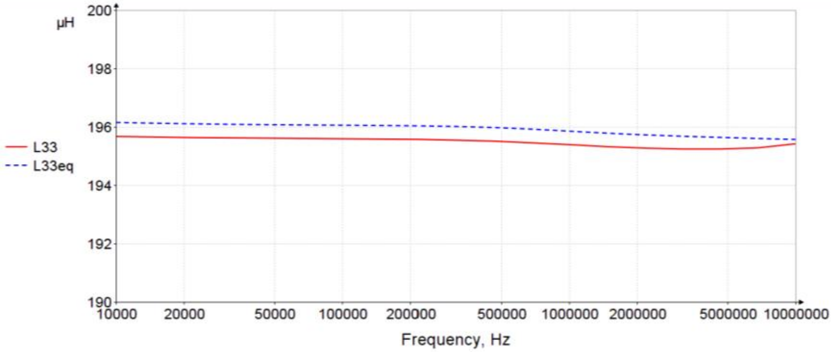




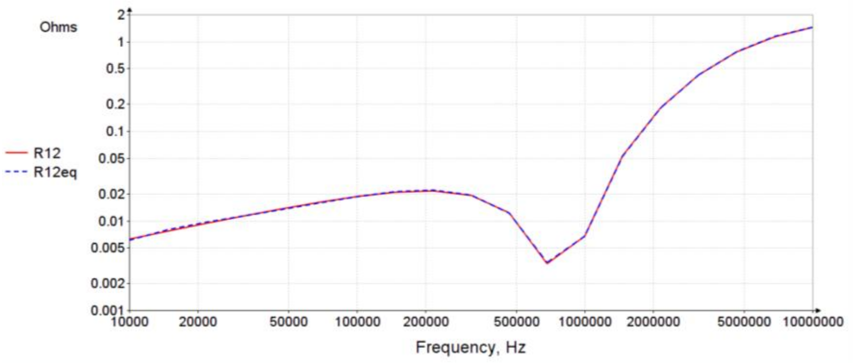


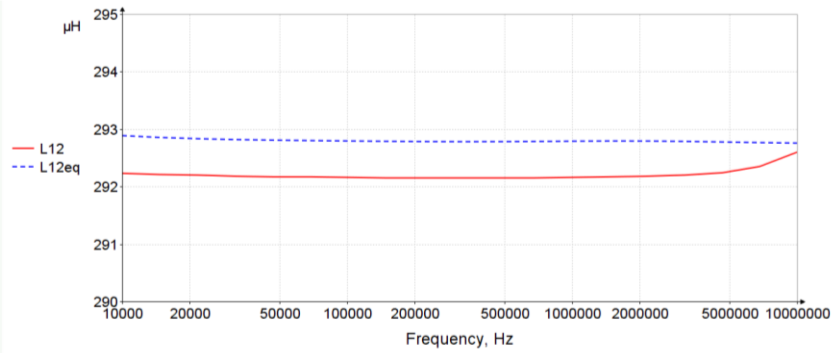


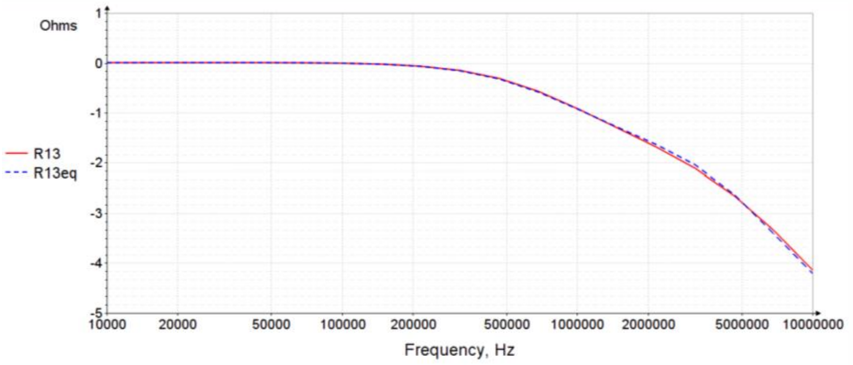


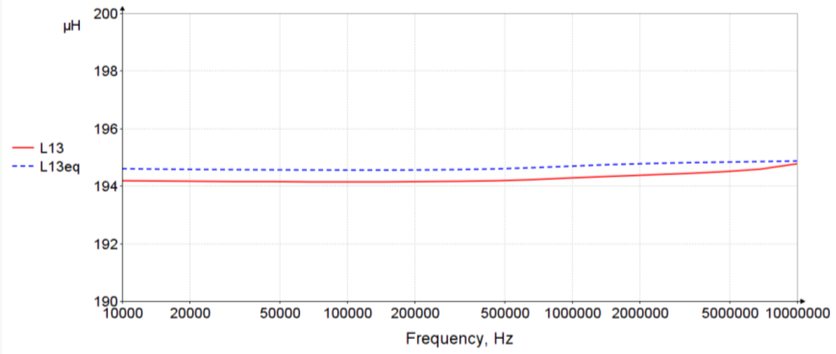


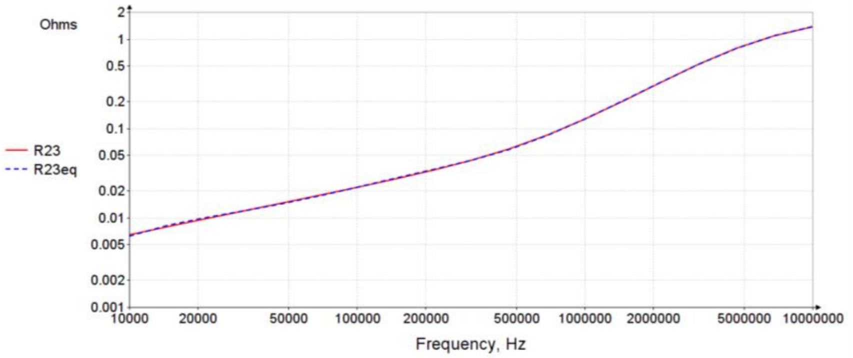
Mutual Resistances and Inductances

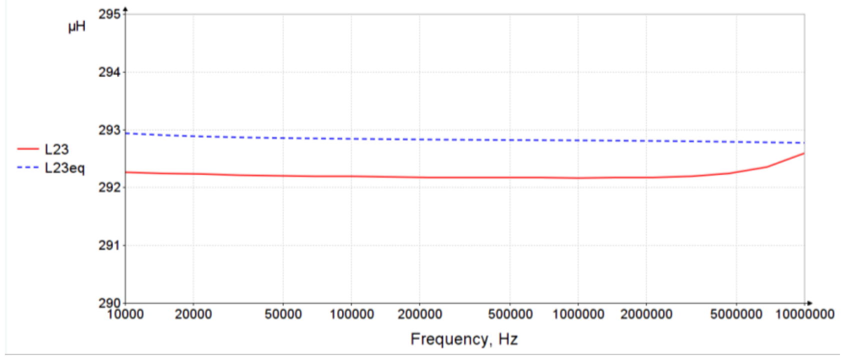




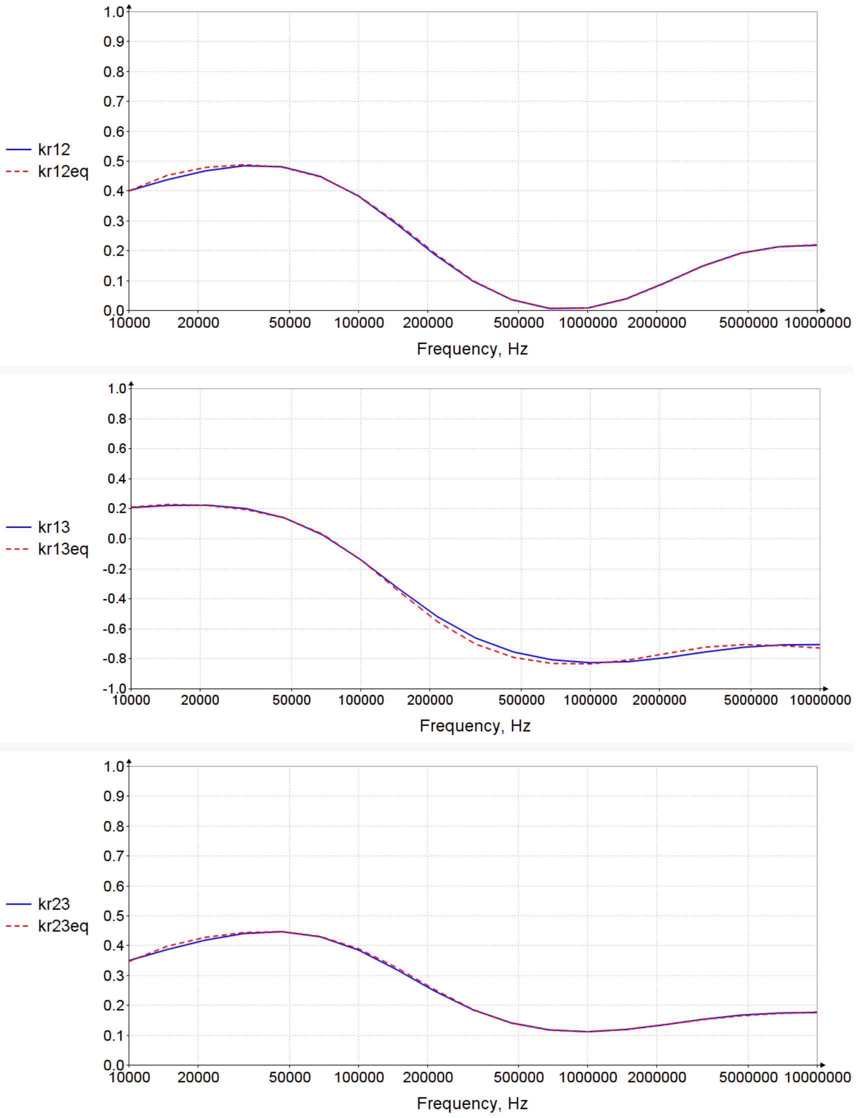




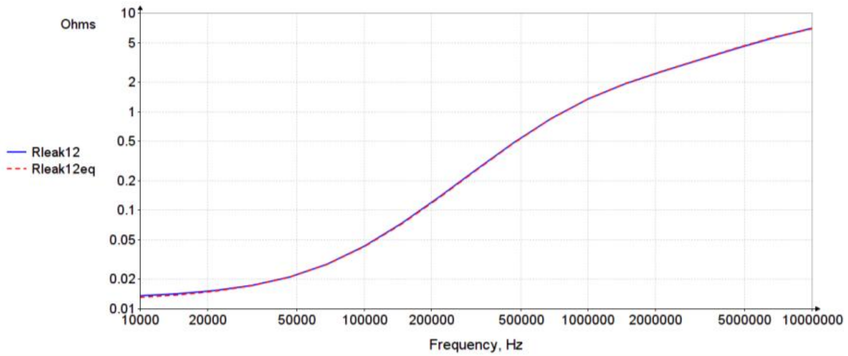


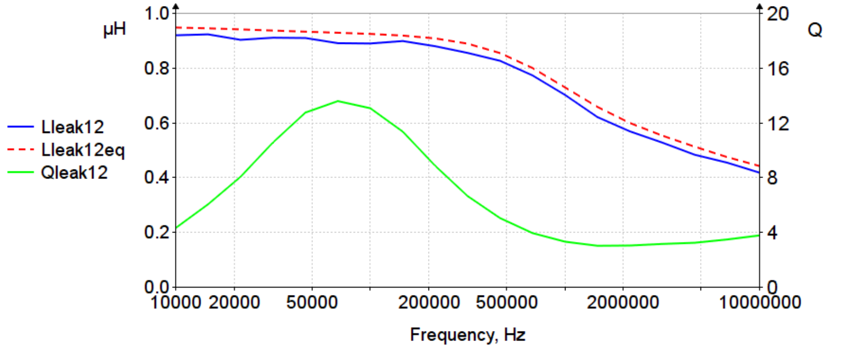


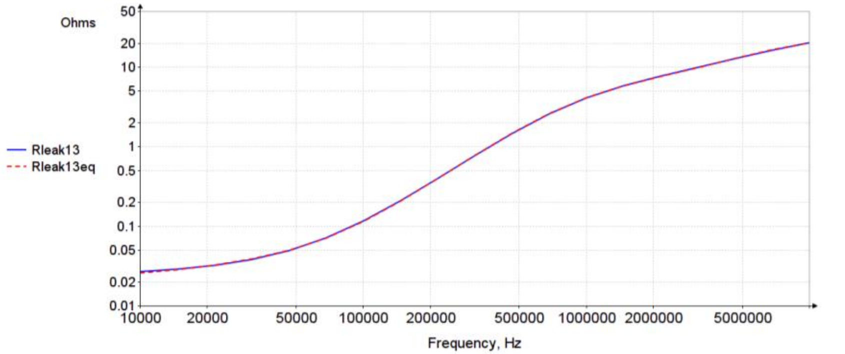
Mutual Resistance Coupling

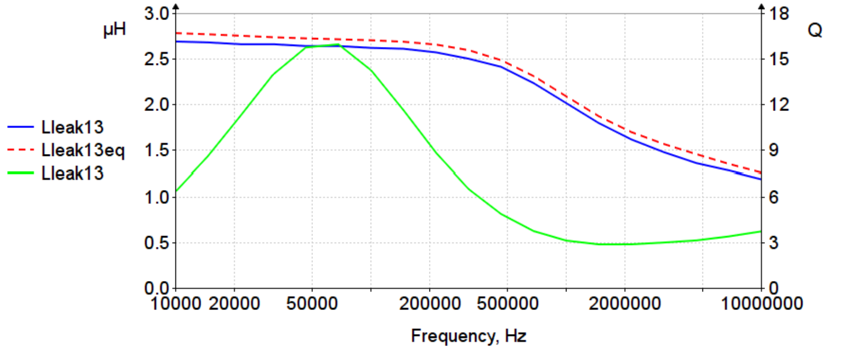


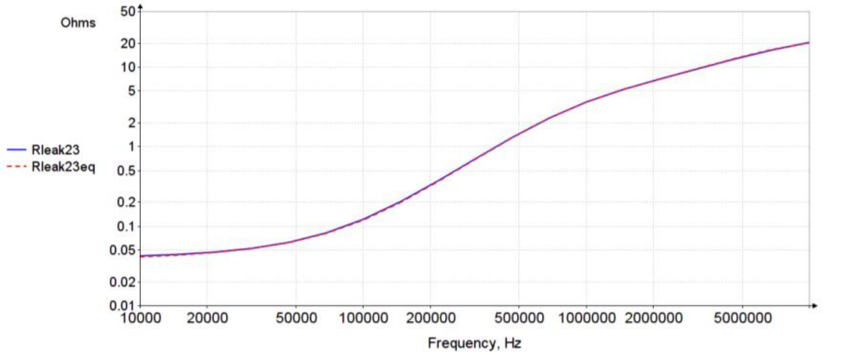
Leakage Resistances and Inductances

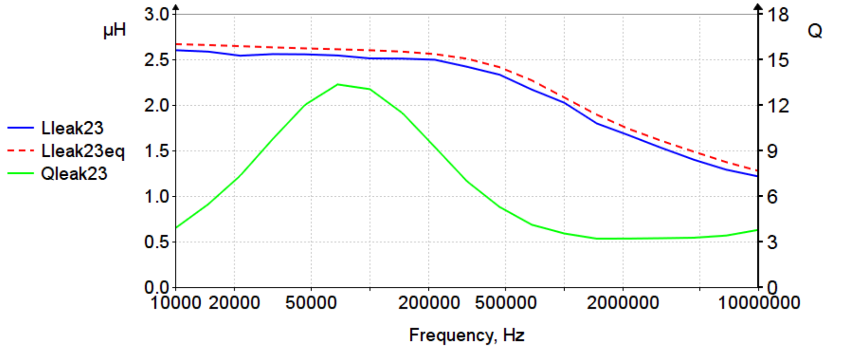




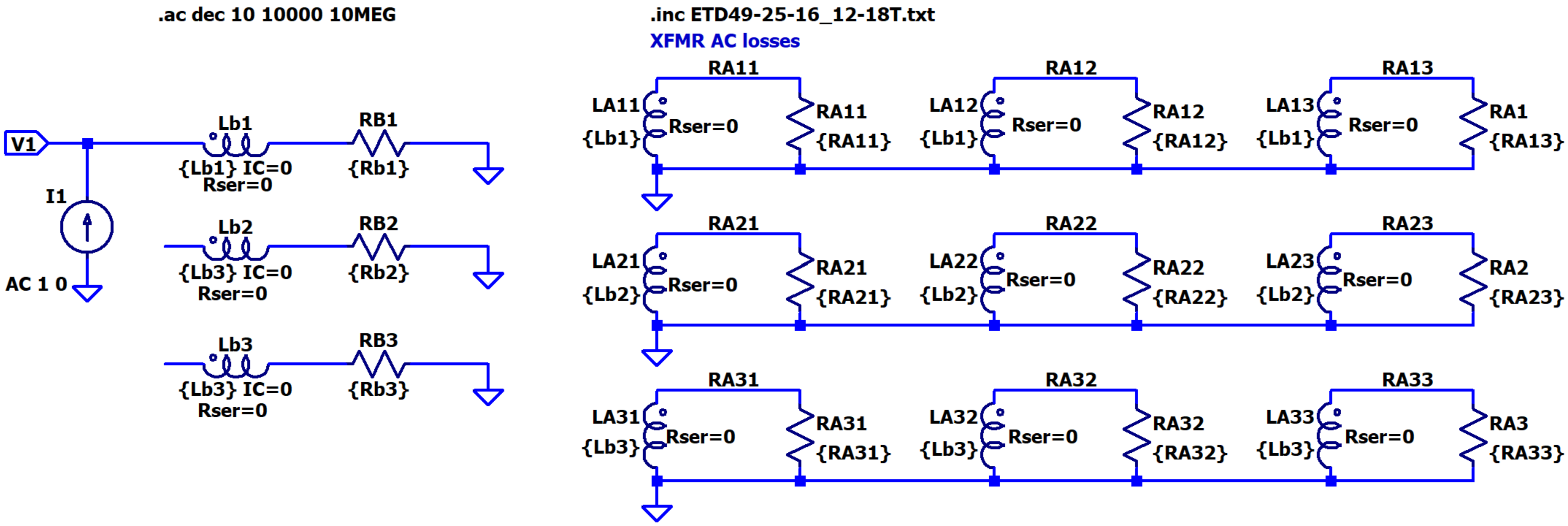




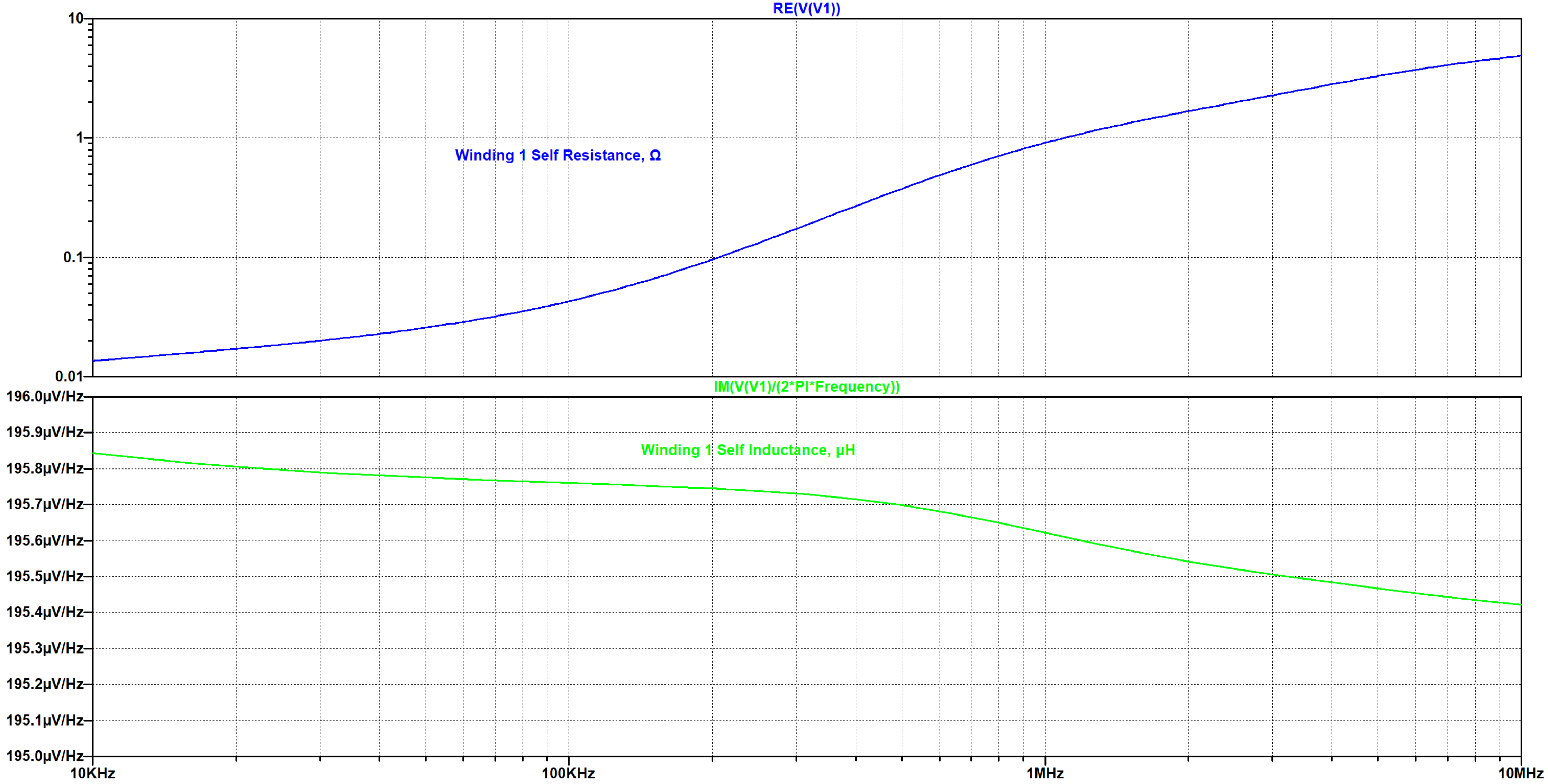




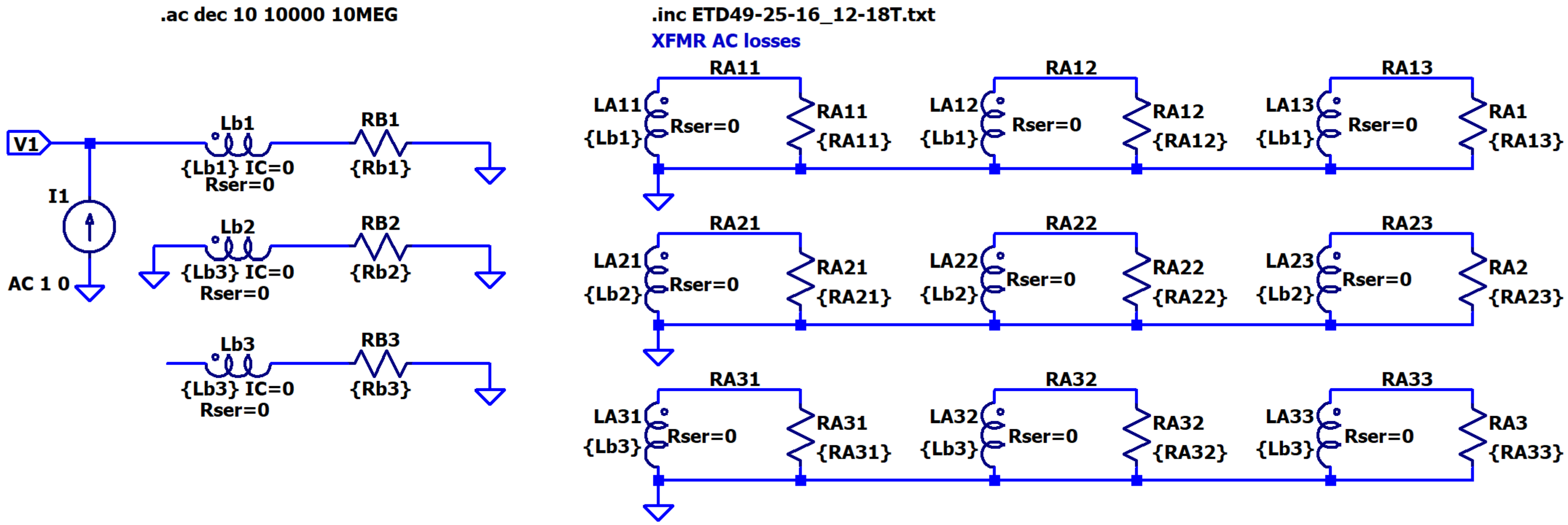
Self Impedance Simulation



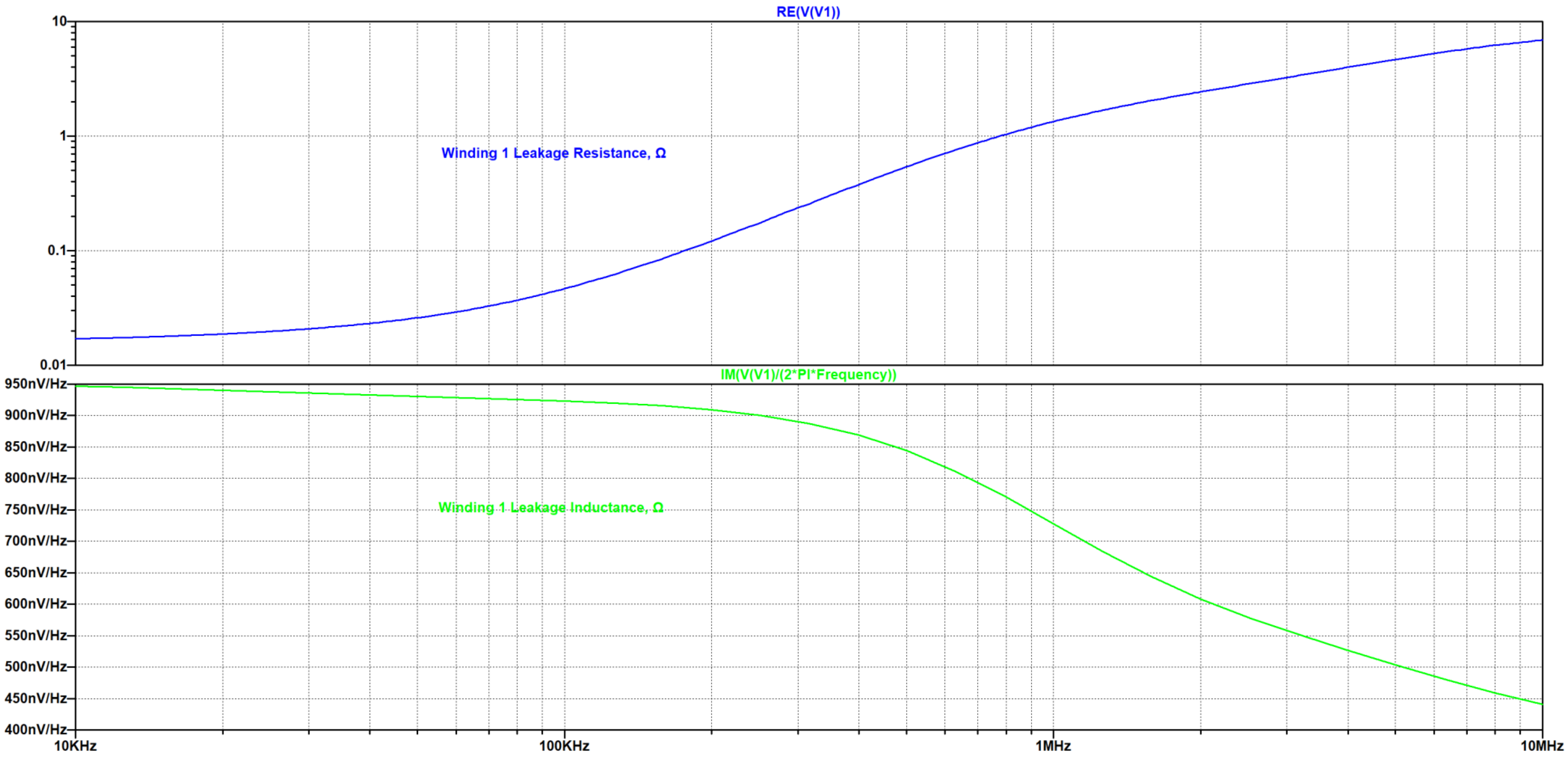
Self Impedance Simulation



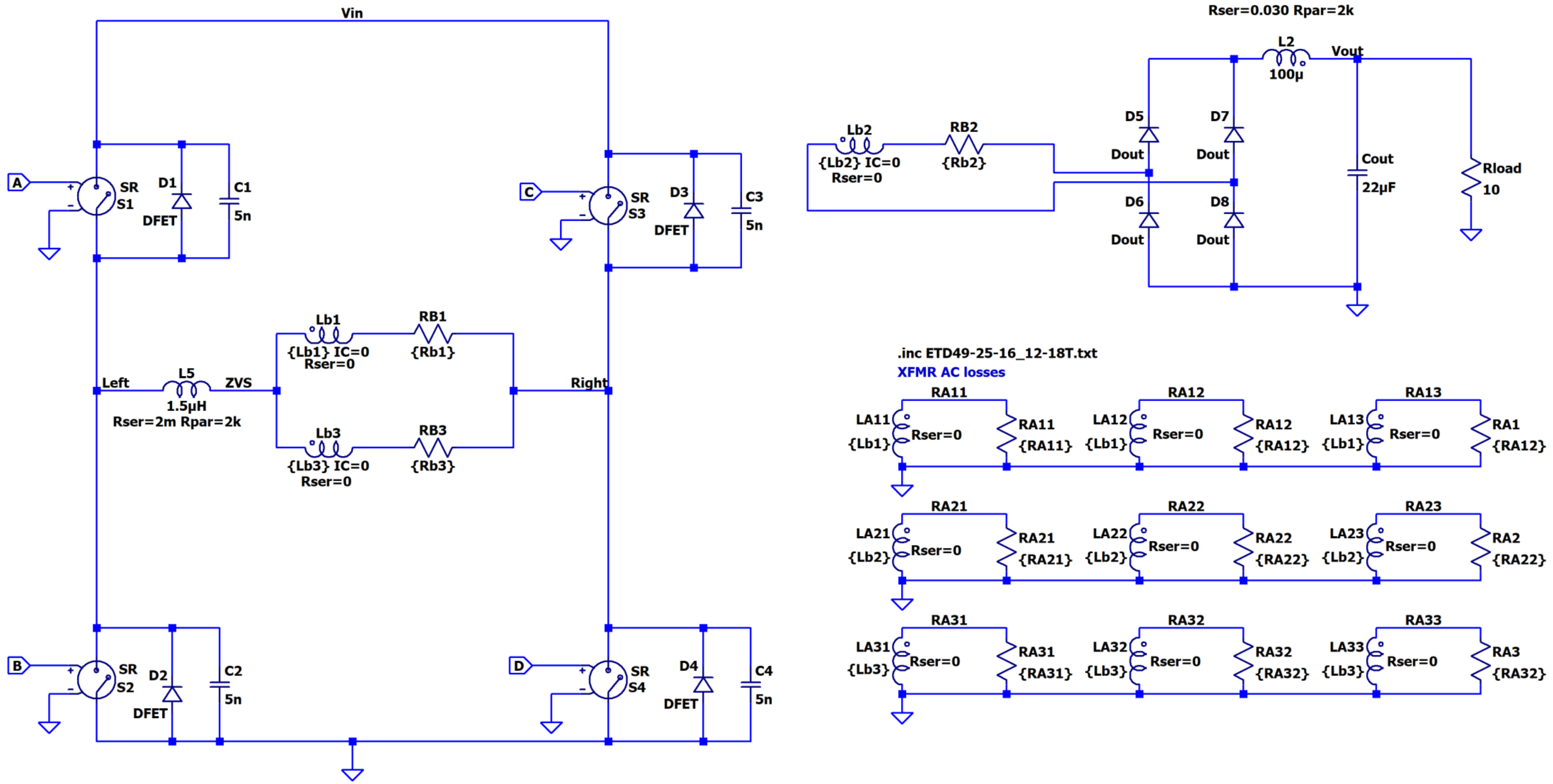
Leakage Impedance Simulation



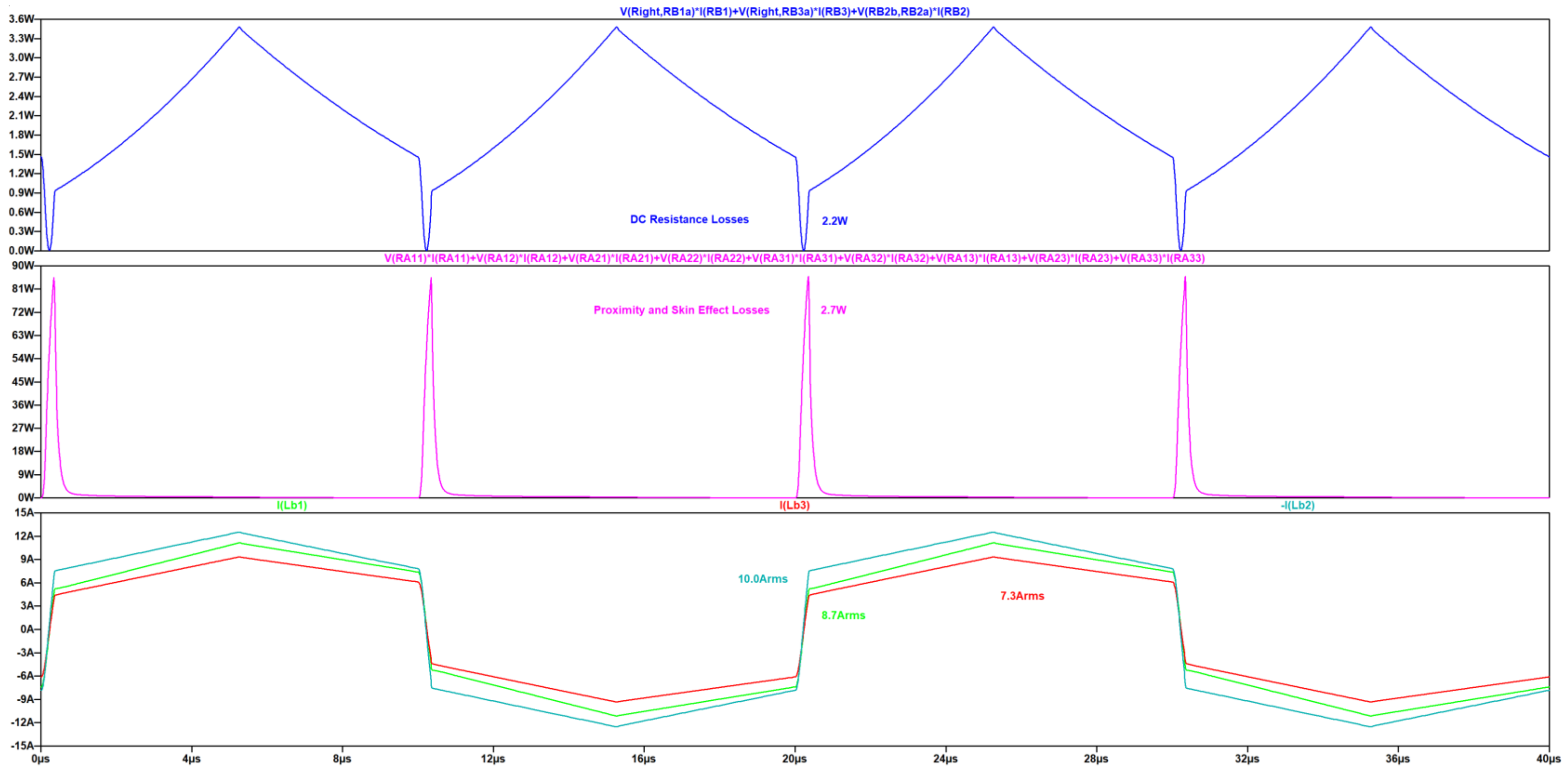
Leakage Impedance Simulation



Phase-Shifted Bridge Simulation



Phase-Shifted Bridge Simulation



- The losses due to the dc and ac resistances are shown above
- The ac losses occur primarily when di/dt is high
- The external ZVS inductor reduces ac losses in the transformer because it reduces di/dt
- Transformer leakage inductances also reduce di/dt, which reduces ac losses

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