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A Treatise on the Theory, Operation, and Application of Transformers

By

The late L. F. BLUME, A. BOYAJIAN, G. CAMILLI,
T. C. LENNOX, S. MINNECI, V. M. MONTSINGER

All of the Transformer Engineering Division of the
General Electric Company, Pittsfield, Massachusetts

SECOND EDITION

One of a series written by General Electric authors for the advancement of engineering practice

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PREFACE TO THE SECOND EDITION

The preface to the first edition of this book has been retained because it states so clearly the origin, purpose, and guiding principles of this second edition as well as of the first.

The progress in certain branches of transformer engineering has been so rapid during the twelve years that have elapsed since the first edition was issued that a second edition was prepared to provide up-to-date information for those who habitually refer to it as well as for those who have become newly interested in the subject.

The chapters that have been changed most are those relating to insulation, thermal characteristics, and the load ratio control practice.

Chapter IX on thermal characteristics includes among other new material (*a*) the latest data on the aging of insulation, (*b*) more accurate methods for the calculation of the temperature rise of water-cooled transformers, (*c*) a rational formula for the temperature rise of windings above the oil temperature, and (*d*) explanation of the latest concepts of permissible overloading of transformers as covered in the *A.S.A. Guides for Operation of Transformers.*

Chapter XV on insulation has been made more complete with the addition of new material, including the latest data on the complete volt-time curve of solid insulation and oil, ranging from approximately one microsecond to infinite time.

While this revision was in progress, L. F. Blume, the editor, died rather suddenly, to the sorrow of his associates; and the revision or rewriting of some of the chapters for which he was responsible was completed by others whose names appear with his.

THE AUTHORS

Pittsfield, Mass.
January, 1951

Mutual Effect of Excitation and Impedance Characteristics. As discussed in the chapter on excitation characteristics, to a very close approximation the shunt or excitation volt-amperes and watts of the transformer are independent of the currents in the windings (as may

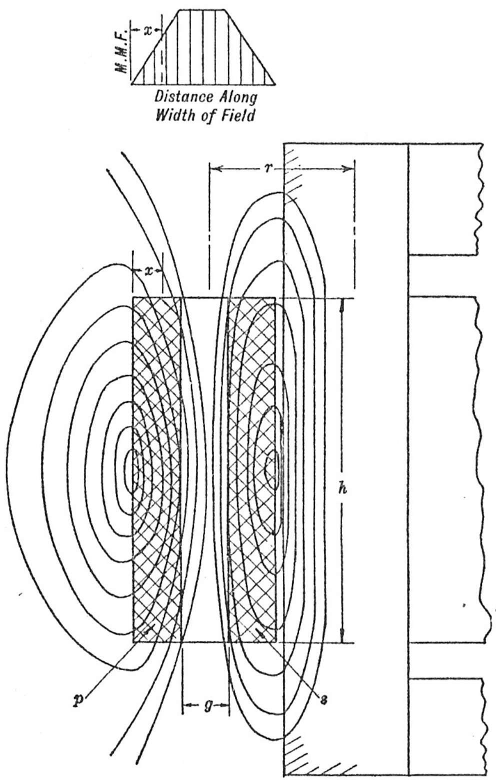
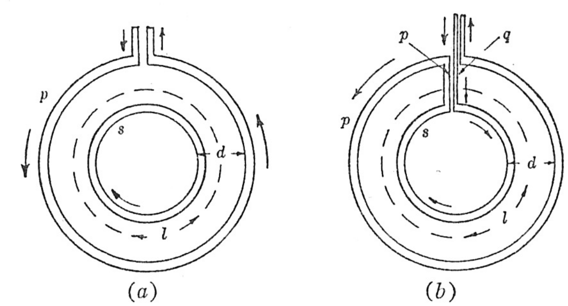


FIG. 16. Leakage field and magnetomotive force diagram of a simple concentric transformer.

be gathered also from the equivalent circuits, especially from that of Fig. 15*b*), and, similarly, the series or impedance volt-amperes and watts of the transformer are substantially independent of the excitation on the windings. Therefore, the total losses of a transformer under any operating condition are conventionally obtained by combining the corresponding excitation and impedance losses as determined independently of each other.

Reactance. *Calculation of Leakage Flux.* Figure 16 represents the longitudinal cross section of a simple transformer having concentric

primary and secondary windings and the leakage flux between them produced by the equal and opposite primary and secondary ampere-turns. This figure could also represent the cross section and magnetic field of two long bus bars carrying equal currents in opposite directions and acting as return circuit to each other. Conceived of as bus bars, p and s would be considered to form a single loop or coil, and the magnetic field would be calculated as that of a single coil. This fact suggests that the two independent solenoids p and s may be considered as return circuits to each other, that is, as forming a single loop. This idea is developed progressively in Fig. 17*a*, *b*, and *c*. The magnetomotive forces and magnetic field of Fig. 17*a* can obviously be replaced by those of Fig. 17*b*. This in its turn can be "developed," that is, straightened out, without materially changing the magnetic



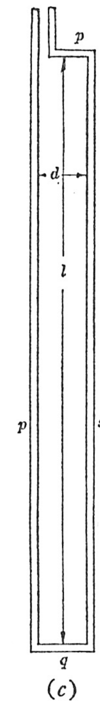


FIG. 17. Illustrating the conception of primary and secondary windings as return circuits to each other, and constituting jointly a single circuit with a single reactance. *a*. End view of actual primary and secondary solenoids. External connections not shown. *b*. An equivalent of *a*: p and q extend the full length of the solenoids. *c*. Showing the curved rectangle of *b* "developed" or straightened out.

field, as in Fig. 17*c*. Thus, we see that p and s form one closed loop with reference to their leakage magnetic field. This conception greatly simplifies the procedure in developing formulas for leakage reactance, permitting the use of single coil or solenoid formulas for the calculation of reactance and leakage flux.

In the upper portion of Fig. 16 is shown the m.m.f. diagram of the distributed ampere-turns due to the load current of the transformer. The m.m.f. at any point x is the total ampere-turns of the windings to the left (or to the right) * of x, giving proper consideration to the algebraic signs of the currents, e.g., the primary considered positive, secondary negative, or vice versa. The m.m.f. producing the field at the gap g is the ampere-turns of p or of s (not the sum of the two), and we may write, for the flux density B_g, in the gap

$$B_g = \frac{4\pi}{10}\frac{NI}{h} \text{ gausses } (h \text{ in cm.}) \quad (14a)$$

$$= 3.2\frac{NI}{h} \text{ lines/sq. in. } (h \text{ in in.}) \quad (14b)$$

in which h is the effective length of the leakage field along the axis of the solenoids.

To a first approximation, we may assume that the leakage flux density (Fig. 16) is uniform in a direction parallel to the coils from one end of the gap to the other; that, when the flux passes beyond the coils, it diverges rapidly, its density is reduced to a low value, most of the flux finds the shortest path to the iron of the yokes or legs and returns with negligible drop; and that, therefore, the reluctance of the outside path of the leakage flux is negligible. Accordingly, we may approximate the proper value of h by the height of the windings.

On this basis, the m.m.f. diagram of Fig. 16 becomes also the flux-density diagram. If the flux density of the gap g is taken as uniform, the total flux ϕ_g in the gap will be the density multiplied by the area of the gap. In inch units,

$$\phi_g = \left(3.2\frac{NI}{h}\right)(2\pi rg) \text{ lines} \quad (15)$$

where r is the mean radius of the gap, and g its width.

The contribution of this flux to the reactance voltage will be $2\pi fN/10^8$ times the flux,

$$e_g = \frac{2\pi fN}{10^8}\left(3.2\frac{NI}{h}\right)(2\pi rg) \text{ volts} \quad (16a)$$

$$= \frac{126f(N^2I)rg}{10^8h} \text{ volts} \quad (16b)$$

$$X_g = \frac{126fN^2rg}{10^8h} \text{ ohms} \quad (17)$$

* The reader may convince himself easily that it is immaterial whether the algebraic summation of the ampere-turns is reckoned from the left up to x or from the right up to x, except for the algebraic sign of the sum.

$$(I^2X)_g = \frac{126f(NI)^2rg}{10^8h} \text{ v-a.} \quad (18)$$

Equation 18 emphasizes the fact that the leakage reactance of a transformer consumes wattless power in the leakage space. This equation has at least two very important advantages over equations 17 and 16 for the purposes of this chapter.

First, it involves the quantity NI which is the same for the primary as for the secondary, regardless of what their number of turns may be, whereas both equations 17 and 16 involve N by itself, which is different for different windings. Thus, the reactance volt-amperes of a transformer are the same whether reckoned in terms of one or the other of the two windings, in contrast with the reactance *ohms* which vary as the square of the turns, and the reactance *volts* which vary as the first power of the turns.

Second, the derivation of equations 16 and 17 implies that the flux links a certain winding and not another. Although it can be shown that, assuming the primary and the secondary windings connected in series opposition, it does not matter whether the flux links the turns on the right or on the left, such discussions tend to become involved. Equation 18, however, can be used without any reference to flux linkages for, if so much reactive power is flowing into the leakage field, there must be that much reactive power input into the exciting winding (in addition to any loads across the secondary), and, therefore, the transformer must display a corresponding effective leakage reactance between its input and output terminals. This conception affords us a method (which we shall call "reactive kv-a. method") whereby equation 18 may be extended readily so as to apply to more general winding arrangements and to non-uniform fields.

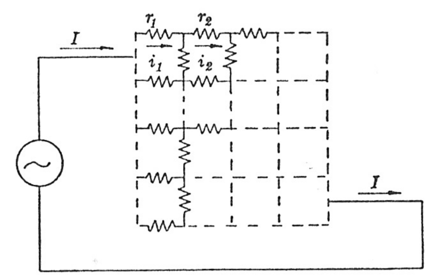


FIG. 18. A general resistance network.

Reactive Kv-a. Method. Figure 18 represents a generalized network through which the line current I finds its way. Assume that the distribution of the currents (i_1, i_2, etc.) in the individual branches of the network is known, and the resistances (r_1, r_2, etc.) of the branches are also given. Required to find the effective resistance $R_{\text{eff.}}$ of the network between its input and output terminals. Obviously, the watts consumed by the network will be $I^2R_{\text{eff.}}$, the $R_{\text{eff.}}$ to be calculated. But the watts consumed by the network may also be obtained by summing up the watts in each branch: *

$$I^2R_{\text{eff.}} = i_1{}^2r_1 + i_2{}^2r_2 + \cdots \tag{19a}$$

$$= \sum_{k=1}^{k=n} (i^2r)_k \tag{19b}$$

and, therefore,

$$R_{\text{eff.}} = \frac{\Sigma(i^2r)_k}{I^2} \tag{20}$$

This general method is equally applicable to the effective reactance of a network as to its resistance, and we may write:

$$I^2X_{\text{eff.}} = \sum_{k=1}^{k=n} (i^2x)_k \tag{21}$$

$$X_{\text{eff.}} = \frac{\Sigma(i^2x)_k}{I^2} \tag{22}$$

The method applies not only to concentrated impedance links but also to distributed fields, as in the present problem, as follows.

If the reactive volt-amperes in the different zones of the magnetic field are known, they can be added together to obtain the total reactive v-a. input into the circuit. Formulas have already been given to calculate the reactive v-a. in any gap in which the density is uniform. Since the reactive v-a. vary as the square of the ampere-turns acting on the gap (see equation 18), it follows that the total effective cross section of the leakage field consisting of zones of different density can be found by adding the weighted cross section of the different zones, the weighting factor of each zone being proportional to the square of the ampere-turns acting on it. The procedure is as follows:

(*a*) *Interleaved-Coil Designs.*

Figure 19*a* represents in cross section the general case of a set of flat

* The alternative to equation 20 would have been to find the effective impedance by simplifying the network by mesh-star and star-mesh transformations, or by determining what are in series, what in parallel, and reducing the network accordingly. These are very laborious in contrast to the simplicity of equation 20. The reactive kv-a. method requires, of course, that the distribution of the currents be known, but this is a relatively easy matter in many of the transformer impedance problems.

primary and secondary coils sandwiched in with each other. The windings may be either shell-type "pancake" coils or core-type disk coils.

If the values of i^2x in the different coils and gaps are known, the total effective reactance can be determined by equation 22. Gap densities

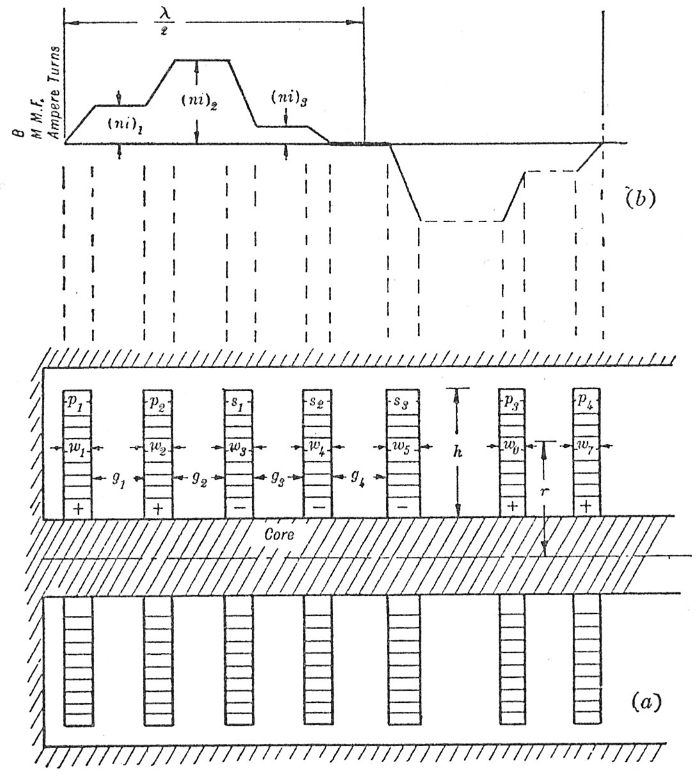


FIG. 19. Arrangement of windings and distribution of leakage m.m.f. of an interleaved design. *a*. Arrangement of coils. *b*. M.m.f. diagram of the leakage field.

differ from each other, and their values follow the m.m.f. diagram (Fig. 19). Since the i^2x of a gap varies with the square of the flux density in the gap, the weighting factor for each gap will be $(ni/NI)^2$, ni being the ampere-turns acting on the gap, and NI the total ampere-turns of the primary (or secondary) winding. Therefore, the reactive v-a. consumed by the gaps will be (from equations 18 and 22),

$$(I^2X)_{\text{gaps}} = \frac{126f(NI)^2}{10^8}\frac{r}{h}\left[\left\{\frac{(ni)_1}{NI}\right\}^2 g_1 + \left\{\frac{(ni)_2}{NI}\right\}^2 g_2 + \cdots\right] \tag{23a}$$

or letting $(ni)_k/(NI)$ be represented by m_k for convenience,

$$(I^2X) = \frac{126f(NI)^2 r}{10^8 h} \sum_{k=1}^{k=n} m_k^2 g_k \text{ v-a.} \tag{23b}$$

and no questions need be asked as to whether the fluxes close one way or another, or whether anything is in series or in parallel.

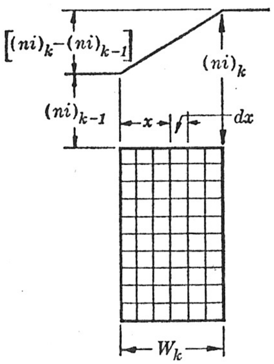


FIG. 20. Variation of leakage field intensity along the width of a coil and its contribution to reactive volt-amperes.

The density of the leakage field within the coils themselves is not constant between the two surfaces of a given current-carrying coil, but at a given surface is the same as the density of the gap adjacent to that surface. Thus, for the kth coil (Fig. 20), the relative density at one surface will be m_k, at the other, m_{k-1}, changing by $(m_k - m_{k-1})$ through the coil width w_k. The relative density at x will be

$$m_{k-1} + \frac{x}{w_k}(m_k - m_{k-1}) \tag{24}$$

and the summation for the reactive v-a. of the whole width of the coil will be

$$(I^2X)_{\text{of } k\text{th coil}} = \frac{126f(NI)^2}{10^8}\frac{r}{h}\left[\int_0^{w_k}\left\{m_{k-1} + \frac{x}{w_k}(m_k - m_{k-1})\right\}^2 dx\right] \tag{25a}$$

Carrying out the simple integration indicated, and then summing up for all the coils,

$$(I^2X)_{\text{of all coils}} = \frac{126f(NI)^2}{10^8}\frac{r}{h}\sum_{k=1}^{k=n}[m_k^2 + m^2_{k-1} + m_k m_{k-1}]\frac{w_k}{3} \tag{25b}$$

In equation 25b the algebraic sign of $(m_k m_{k-1})$ must be entered correctly, as the magnetomotive force acting on a gap may be either positive or negative. Of course, m_k^2 and m^2_{k-1} will always be positive.

The summation for the gaps (indicated by equation 23b) and that for the coils (indicated by equation 25b) may be combined, as in equations 26b and 27b, and conducted conveniently in tabular form. So we have, for interleaved (flat-coil) designs (Fig. 19a),

$$(I^2X)_{\text{total}} = K\frac{126f(NI)^2}{10^8}\frac{r}{h}D \tag{26a}$$

$$D = \sum_{k=1}^{k=n}[m_k^2 g_k + (m_k^2 + m^2_{k-1} + m_k m_{k-1})w_k/3] \tag{26b}$$

in which K* is introduced as a correction factor to compensate for various approximate assumptions made, and D will be seen to be the effective equivalent width of the leakage field as a uniform field with full primary (or secondary) ampere-turns acting on it.

(b) *Cylindrical-Coil Designs* (Fig. 16).

In concentric transformers, the radius r is not the same for all the coils and gaps, and r therefore must be included in the summation for D as follows. Letting r_k stand for the mean radius of the kth gap, and r'_k for that of the kth coil,

$$(I^2X)_{\text{total}} = K\frac{126f(NI)^2}{10^8 h}(rD) \tag{27a}$$

$$(rD) = \sum_{k=1}^{k=n}[m_k^2 r_k g_k + (m_k^2 + m^2_{k-1} + m_k m_{k-1})r'_k w_k/3] \tag{27b}$$

Leakage Field Non-Uniform in a Longitudinal Direction. It may be seen from the foregoing that the reactance calculation of a transformer is rather involved. Yet, if equations 26 and 27 were all there was to it, transformer designers would be contented. But these equations are only approximations, based on the assumptions that the flux density is uniform along the flux lines within the coil boundaries (which is not exactly right) and zero outside of the coil boundaries (which also is not exactly right).

The degree of approximation involved in these equations is a function of the ratio $2h/\lambda$, λ being the effective width of the field per wave length† of the m.m.f. diagram (Fig. 19): the larger this ratio, the closer is the approximation. In concentric transformers, with a $2h/\lambda$ ratio of 10–20, the approximation is within commercial tolerance limits. In shell-type and in interleaved disc designs, with a $2h/\lambda$ ratio of the order of 1–4, this simple approximation is not acceptable, and some correction factor has to be applied. Even in concentric designs, if the coils are extremely short compared with usual proportions, the m.m.f. drop in the outside path cannot be ignored.

* See equations 29, 30, and 31.

† In symmetrical designs, the m.m.f. diagram repeats itself like a standing wave in space. One "wave length" includes a positive and a negative loop.

Non-uniformity along the flux lines is found to be a much more difficult problem than non-uniformity transverse to the flux lines, because the relative densities in the latter case can be approximated by the m.m.f. diagram (Fig. 19) and proper allowances made as in equations 26 and 27, but no simple guide is available to the former.

Still more serious difficulties arise if the primary and secondary coils are of unequal length ($h_2 \neq h_1$), or if either one of the coils is non-uniform, or if appreciable taps are taken out from within the windings.

Some of the methods that have been proposed for the solution of one or another class of these problems follow.

Empirical Correction Factors. Results within engineering requirements can be obtained by empirical correction factors for a customary range of proportions, based on a large number of data, but this method has the limitation that correction curves so obtained cannot be safely extrapolated, and they fail when the design proportions are unusual.

It must be realized further that this method is suitable only for simple symmetrical designs with primary and secondary coils of equal length h.

Use of Self and Mutual Inductance Formulas. Equation 28:

$$X_{12} = X_1 + X_2 - 2M_{12} \tag{28}$$

defines the leakage reactance X_{12} in terms of the self and mutual magnetizing reactances X_1, X_2, and M_{12}, and, therefore, if these latter can be computed, the former follows. Electrical literature, especially various bulletins of the National Bureau of Standards,* contains many valuable formulas for self and mutual inductance calculations.

Although mathematically fundamental, this method has several practical difficulties and limitations:

(*a*) Since, in this method, leakage inductance has to be obtained as the difference of two quantities very nearly equal to each other, that is, as

$$(X_1 + X_2) - (2M_{12})$$

the self and mutual inductance values have to be calculated correct to four or more significant figures to obtain the leakage inductance correct to two significant figures. Such calculations therefore cannot be made with ordinary slide rules.

(*b*) Self and mutual inductance formulas which take into account coil thickness are very laborious.

(*c*) The presence of iron adds further factors to be taken into account. In the majority of simple symmetrical designs, the core has very little effect on the leakage reactance, and air-core self and mutual inductance formulas can be used for them if desired. But simple symmetrical designs do not require such laborious methods, and the very cases in which the reactance cannot be calculated by simple methods are generally those whose reactance is influenced materially by the presence of the iron core in the return magnetic circuit of a component of the leakage field; and, therefore, the presence of the core has to be taken into account, and air-core inductance formulas are not adequate. The presence of the core can be taken into account by the so-called method of images discussed below, but it will be appreciated that the core is not a smooth continuous mirror, and therefore again simplifying approximations have to be made.

* See more especially *Scientific Paper 169* of the National Bureau of Standards.

(*d*) Finally, if the primary and secondary windings cannot be considered as a single uniform continuous coil (and they cannot be so considered in shell-type or in interleaved disk-coil designs, or in concentric designs having sections tapped out from the middle of the windings, or in any design with varying coil dimensions), then, self and mutual inductance values have to be calculated not only for a large number of subdivisions of the windings, but also between the images of each of these coils and the real coils. This labor mounts very rapidly with increasing number of coils, and inaccuracies due to use of limited number of significant figures in the various terms tend to increase.

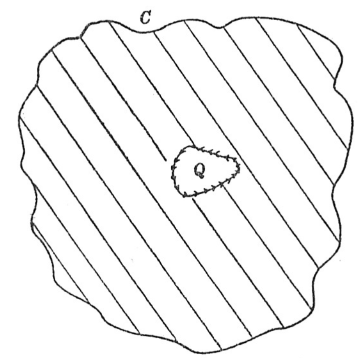


FIG. 21. C represents the cross section of a conductor of any shape carrying a current. Q is a spot of any shape on the cross section. The algebraic sum of the magnetomotive force drops, along the periphery of Q, is equal to 4π times the amperes flowing through the spot perpendicular to plane of the spot. Currents outside the spot do not alter this relationship.

For these reasons, self and mutual inductance formulas are little used in leakage reactance computations.

Application of Field Equations. (*a*) Magnetic flux always forms a closed loop, in contrast to electrostatic flux which radiates from one point and terminates at another.

(*b*) If we consider a small spot in the magnetic field, say Q in Fig. 21, and sum up the total m.m.f. (ampere-turns) consumed by the flux along the perimeter of the spot, this must equal the amperes flowing through the spot perpendicular to the spot.

The presence of a current flow in the neighborhood of the spot but outside of it will affect the flux at various points within the spot, but the m.m.f. of this will be found to cancel out when summed up around the spot.

These two elementary facts, when stated in mathematical form * and solved with terminal conditions incorporated, give us a formula for the distribution of the magnetic field and hence a formula for reactance.

To carry out such an analysis for each individual problem would require a prohibitive amount of labor, but if the problem could be solved in a generalized form it might be made to furnish correction formulas broadly applicable to all transformers. This was attempted many years ago by Dr. Rogowski,[1] and a solution was obtained by him applicable to symmetrical shell-type and interleaved disk-coil designs.

When iron is absent, or far enough from coils to make its effect negligible, the Rogowski correction factor to our formulas 26 and 27 is

* The two statements of paragraphs (*a*) and (*b*) take the mathematical forms

(*a*)
$$\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} = 0 \qquad (1)$$

(*b*)
$$\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} = \frac{4\pi}{10} i_z \qquad (2)$$

where B_x and B_y are the flux densities in the x and y directions, respectively, and i_z is the current density at the spot (x, y) perpendicular to the x-y plane, and $4\pi/10$ converts the current into m.m.f. units. Of course, in regions where there is no current, i_z in (2) is equated to zero.

If we represent by ϕ_z the total flux having the axis z and surrounding the current i_z, then

$$B_x = \frac{\partial \phi_z}{\partial y} \qquad (3) \qquad\qquad B_y = \frac{-\partial \phi_z}{\partial x} \qquad (4)$$

ϕ_z is called the "vector potential" of the flux density B.

It will be seen that (3) and (4) are consistent with (1), as their substitution renders (1) identically zero. Their substitution into (2) gives

$$\frac{\partial^2 \phi_z}{\partial x^2} + \frac{\partial^2 \phi_z}{\partial y^2} = \frac{4\pi}{10} i_z \qquad (5)$$

The solution of (5), involving exponential and harmonic series, gives the value of ϕ_z, and the integration of the product of ϕ_z with i_z throughout the x-y plane gives the total flux linkages (and hence the inductance) per unit length of the windings in the direction (z) of the current, that is, for unit length along the mean perimeter of the turns.

[1] Superior numbers refer to the "References" which are listed at the end of the chapter.

$$K = 1 - \frac{1 - \epsilon^{-2\pi h/\lambda}}{2\pi h/\lambda} \qquad (29)$$

λ being the wave length of the m.m.f. wave (Fig. 19). When the effect of the core on reactance is to be taken into account, the correction factor depends on whether the design is core or shell type as follows:

For the core-type disk windings:

$$K = 1 - \frac{1 - \epsilon^{-2\pi h/\lambda}}{2\pi h/\lambda}\left[1 - \frac{\epsilon^{-4\pi b/\lambda}}{2}(1 - \epsilon^{-2\pi h/\lambda})\right] \qquad (30)$$

in which b is the distance between coil and core leg.

For shell-type designs:

$$K = 1 - \frac{1 - \epsilon^{-2\pi h/\lambda}}{2\pi h/\lambda}\left[1 - \frac{\epsilon^{-4\pi b/\lambda}}{2}(1 - \epsilon^{-2\pi h/\lambda}) \times \left\{\frac{L_2}{L} + \frac{L_1}{L}(1 + \epsilon^{-2\pi(b'-b)/\lambda})\right\} - \frac{L_1}{L}\epsilon^{-2\pi(h+2b+2b')/\lambda}\right] \qquad (31)$$

in which L is the mean coil perimeter, L_1 that part of the perimeter which has iron on both sides, L_2 the rest of the coil perimeter, and b and b' distances from iron to coil on the two sides.

A glance at the equations and transformations of the original Rogowski article [1] reveals at once how formidable is the undertaking to calculate the reactance of an arbitrary field by such equations and how necessary to make many simplifying assumptions. The Rogowski correction, having been developed with the aid of such approximations, gives reasonably good results only with those designs that conform to the simplifying assumptions made, and fails in others. These "others" include the important classes of designs with unequal primary and secondary lengths (h_1, h_2), coils interrupted by taps (h_1 and h_2 interrupted by breaks of considerable length), high-voltage transformers with non-uniform ampere-turn distribution due to grading of insulation (especially in grounded designs), etc.

Resolution of the Leakage Field into Axial and Transverse Components. The calculation of certain types of difficult reactance problems is greatly simplified by a method due to H. O. Stephens,[2] which consists of resolving the magnetic field into two components, one axial and the other radial, best explained by a simple example.

Figure 22 shows a primary and a secondary coil of unequal length. They may be conceived of as either a concentric or an interleaved design, but, to assist in visualization, let them be assumed concentric windings. The coils are shown with either a plus or a minus sign to indicate that their ampere-turns are opposed.

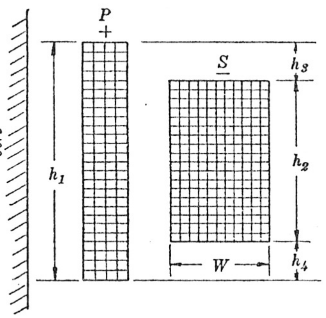


FIG. 22. Concentric design with primary and secondary of unequal length.

The greater the disparity between h_1 and h_2, the greater is the difficulty of calculating the reactance of such a design accurately by elementary formulas.

According to the Stephens scheme, a simplified complete equivalent of this is shown in Fig. 23, comprising two components.

Component I. Axial or Concentric Component, X_I. Figure 23*a* shows this component, comprising coils 1 and 2. Coil 1 is identical with P of Fig. 22. Coil 2 (that is, $2a + 2b + 2c$) is one uniform continuous coil of the same width w as S, but of the same length h_1 as P. Coil 2 has the same total ampere-turns and the same ampere-turns per inch distribution as coil 1; and, as the two coils are of equal length and symmetrical, their leakage reactance can be calculated with far closer approximation than that of Fig. 22 directly. The purpose in showing coil 2 (Fig. 23*a*) in

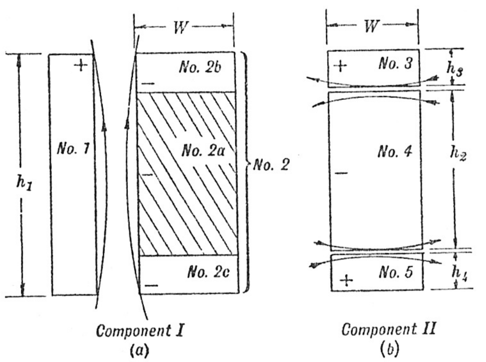


FIG. 23. Equivalent representation of Fig. 22 in terms of two components. *a*. Concentric component with coils of equal length. *b*. Interleaved component.

three sections is merely to indicate its relationship to the original secondary (which has the configuration of coil 2*a*) and to fictitious windings 2*b* and 2*c*.

Since coil 2 has the same ampere-turns as 1, and, hence, the same as P or S, but is h_1/h_2 times as long as S, it follows that the ampere-turn density (that is, ampere-turns per inch) in coil 2 are h_2/h_1 fraction of that in S.

Component II. Transverse or Interleaved Component, X_{II}. This component comprises coils 3, 4, and 5 (Fig. 23*b*). Their dimensions are indicated in terms of the dimensions of the coils of Fig. 22. The ampere-turns of these coils are such that, when the $(3 + 4 + 5)$ ensemble is superposed on $(2a + 2b + 2c)$ ensemble of Fig. 23*a*, coil 3 neutralizes completely the ampere-turns of 2*b*, coil 5 that of 2*c*, and coil 4 *adds* to that of 2*a* and makes it identical with that of the actual secondary S. When this condition exists, then, the complete equivalence of Fig. 23 to Fig. 22 is established. This condition is realized if the various ampere-turns are as follows:

$$\left.\begin{aligned} (ni)_{2a} &= -(NI)_p h_2/h_1 \\ (ni)_{2b} &= -(NI)_p h_3/h_1 \\ (ni)_{2c} &= -(NI)_p h_4/h_1 \\ (ni)_3 &= +(NI)_p h_3/h_1 \\ (ni)_4 &= -(NI)_p (h_3 + h_4)/h_1 \\ (ni)_5 &= +(NI)_p h_4/h_1 \end{aligned}\right\} \quad (32)$$

It will be seen that the primary and secondary ampere-turns of each one of the two component systems add up to zero.

The reactance of this equivalent system will be seen to be the sum of X_I from Fig. 23*a*, X_{II} from Fig. 23*b*, and their mutual induction $M_{I\text{-}II}$:

$$X_{PS} = X_I + X_{II} \pm 2M_{I\text{-}II} \quad (33)$$

An examination of Fig. 23*a* and *b* will show that the leakage field of component II is in quadrature with that of component I, and that therefore mutual reactance between the two component systems will be either zero or one of a second order of magnitude arising from lack of perfect symmetry and from curvature of flux lines. Accordingly, as an approximation, $M_{I\text{-}II}$ may be ignored and the above equation written as

$$X_{PS} \cong X_I + X_{II} \quad (34)$$

As X_I is a symmetrical concentric reactance, and X_{II} a normal interleaved reactance, both calculable by appropriate standard formulas, the calculation of X_{PS} is thereby greatly simplified.

sistance R interposed in series between the input and the output terminals. The single leakage reactance is the sum of the primary and secondary leakage reactances, and the single resistance the sum of the primary and secondary resistances, all reduced to the basis of the circuit in terms of which the voltage regulation is to be expressed.

Based on Fig. 26, the following approximate formula gives the voltage regulation very closely for most cases:

$$\% \text{ Regulation} = kp\%IR + kq\%IX + \frac{(kp\%IX - kq\%IR)^2}{200} \quad (46a)$$

in which k is the actual load as a fraction of the base load on which the values of $\%IR$ and $\%IX$ have been based; p is the power factor (or cos θ) of the load; and q is the reactance factor (or sin θ) of the load. Power factor is a positive number, but the reactance factor may be positive or negative, depending on whether the load is lagging or leading, respectively:

$$q = \pm\sqrt{1 - p^2}$$

In equation 46a, the algebraic sign of q for lagging loads is positive.

If the voltage regulation and the resistance and reactance drops are expressed in per-unit values, the formula becomes

$$\text{Regulation (per unit)} = kpR + kqX + \frac{(kpX - kqR)^2}{2} \quad (46b)$$

The only difference in the two formulas will be seen to be in the denominator of the fraction—200 in one case, 2 in the other.

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2. "Transformer Reactance and Losses with Non-uniform Windings," H. O. Stephens, *Electrical Engineering,* February, 1934, pp. 346–349.
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CHAPTER V

IMPEDANCE CHARACTERISTICS OF MULTICIRCUIT TRANSFORMERS

By A. Boyajian

Necessity for Multicircuit Transformers. Since the purpose of a simple (two-winding) transformer is to interconnect two circuits of different voltage rating, it is obvious that a multicircuit transformer would naturally be considered when three or more circuits are to be interconnected. Such interconnection could be accomplished by a plurality of two-circuit transformers also, but only at greater expense and lower efficiency. Aside from the insulation of circuits of different voltages from each other, multicircuit transformers are also utilized to introduce certain impedances between circuits for control of load division or of short-circuit currents.

An obvious occasion for a multicircuit transformer is when it is desired to feed a distribution system from two transmission circuits of different voltages, whether for continuity of service or economy of operation. A slightly different, but essentially similar, case is that of a generating system feeding two or more outgoing transmission circuits of different voltages. The necessity for handling an auxiliary load, such as a synchronous condenser or a local load, at a voltage different from that of either the primary or the secondary voltage, generally calls for a three-winding transformer. Split-winding generators are another group calling for multicircuit transformers, with the interesting feature that they require two separate primary windings of the same voltage. A similar case is the occasional need to subdivide the secondary load into two separate secondary windings for the purpose of reducing the short-circuit kv-a. due to faults, through manipulation of the transformer reactances without serious derangement of the voltage regulation of the system. Testing transformers with potential coils for direct voltmeter connection will be recognized as another special class of multicircuit transformers with their own peculiar problems. Auto-transformers with tertiary or other insulated windings are still other examples of multicircuit transformers. Finally, load ratio control circuits frequently involve complicated multicircuit problems.