Leakage Inductance

Bryce Hesterman

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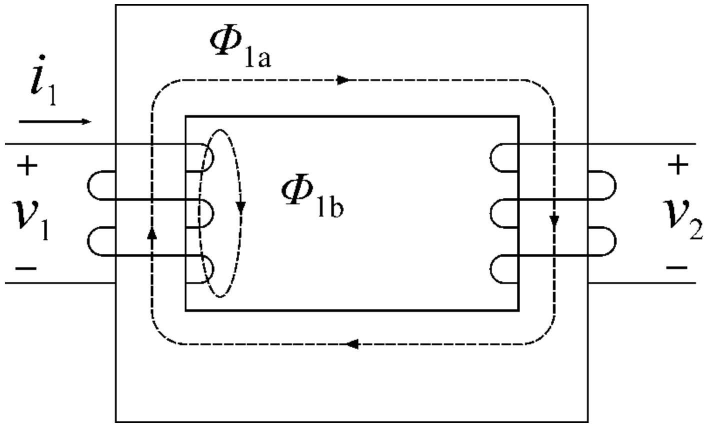
Overview

- This presentation reviews the concepts of leakage impedances which can be represented in terms of leakage inductances and leakage resistances
- Leakage inductances are known to vary with frequency, but this presentation shows how they also vary with time or pulse width
- Only resistive and inductive effects are considered in the presentation

Magnetic Coupling Review



- Two windings are coupled when some of the magnetic flux produced by currents flowing in either of the windings passes through both windings
- Only part of the flux produced by a current in one winding reaches other windings
- Flux which doesn't pass through both windings is called leakage flux
- Magnetic coupling can be modeled in terms of self and mutual impedances

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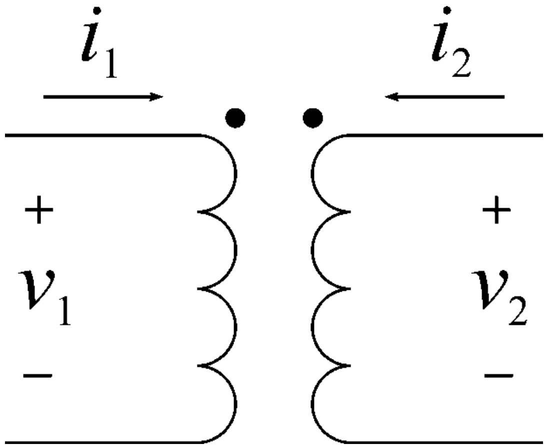
Self and Mutual Impedance Equations



$Z_{11} = \text{Winding 1 Self Impedance}$

$Z_{22} = \text{Winding 2 Self Impedance}$

$Z_{12} = Z_{21} = \text{Mutual Impedance}$

$R_{12} = R_{21} = \text{Mutual Resistance}$

$L_{12} = L_{21} = \text{Mutual Inductance}$

$$v_1 = Z_{11} i_1 + Z_{12} i_2$$
$$= (R_{11} + j\omega L_{11}) i_1 + (R_{12} + j\omega L_{12}) i_2$$

$$v_2 = Z_{21} i_1 + Z_{22} i_2$$
$$= (R_{21} + j\omega L_{21}) i_1 + (R_{22} + j\omega L_{22}) i_2$$

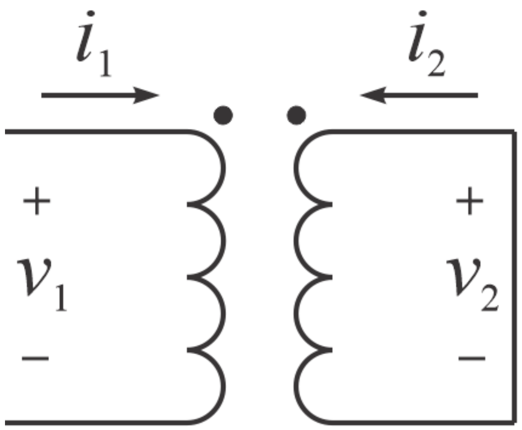
Impedance Matrix Equation for N Windings

$$\begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1N} \\ Z_{21} & Z_{22} & \cdots & Z_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{N1} & Z_{N2} & \cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_N \end{bmatrix}$$

- A set of coupled windings can be modeled with a matrix equation that relates frequency-domain winding voltages and currents with an impedance matrix
- The values of impedance matrix elements can be obtained through FEA simulations or from measurements
- The impedance matrix values vary with frequency

Basic Definition of Leakage Impedance

Z_{leak_mn}



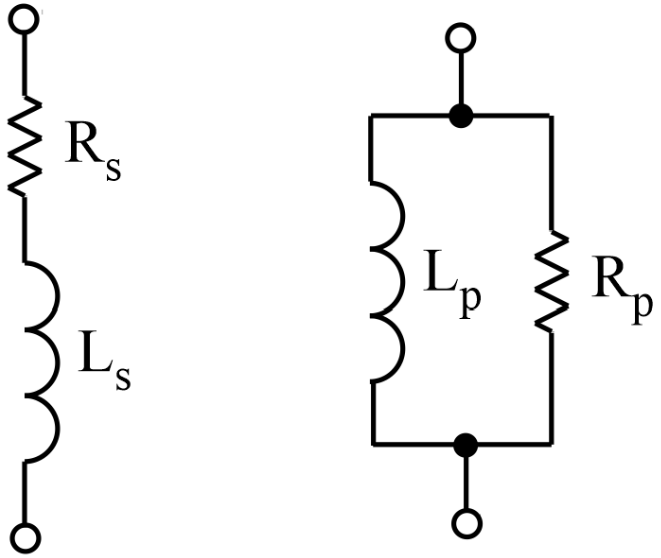
Winding m

Winding n shorted

$$Z_{leak\_mn} = Z_{mm} - \frac{{Z_{mn}}^2}{Z_{nn}}$$

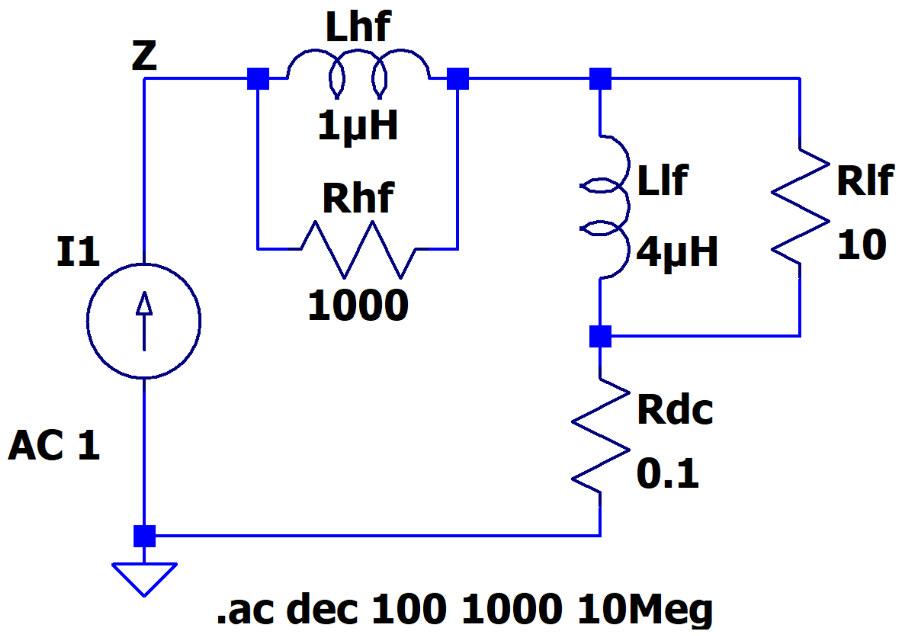
- Leakage impedance is the impedance measured at one winding when another winding is shorted
- Leakage impedances are a function of self and mutual impedances as shown in the equation above
- Consequently, leakage impedances are a property of a pair of windings and generally can't be split up and assigned to individual windings when there are more than two windings

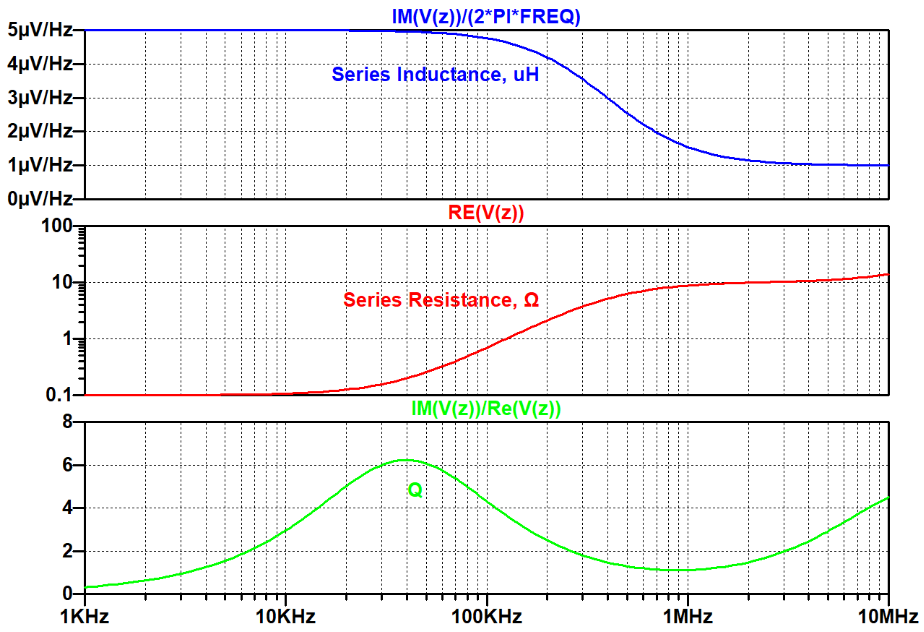
Inductance and Resistance Measurement Options



- LCR meters and network analyzers typically measure the real and imaginary parts of the impedance and then convert that data into series or parallel representations

Measurement of Series Inductance and Resistance





- The impedance of leakage inductances between closely-coupled windings often behave like this circuit where the Q is relatively low
- The series inductance transitions from 5 μH (Lhf + Llf) at low frequencies to about 1 μH (Lhf) at high frequencies due to Rlf
- The parallel resistors represent copper losses
- The core losses are minimal for leakage inductances because one winding is shorted

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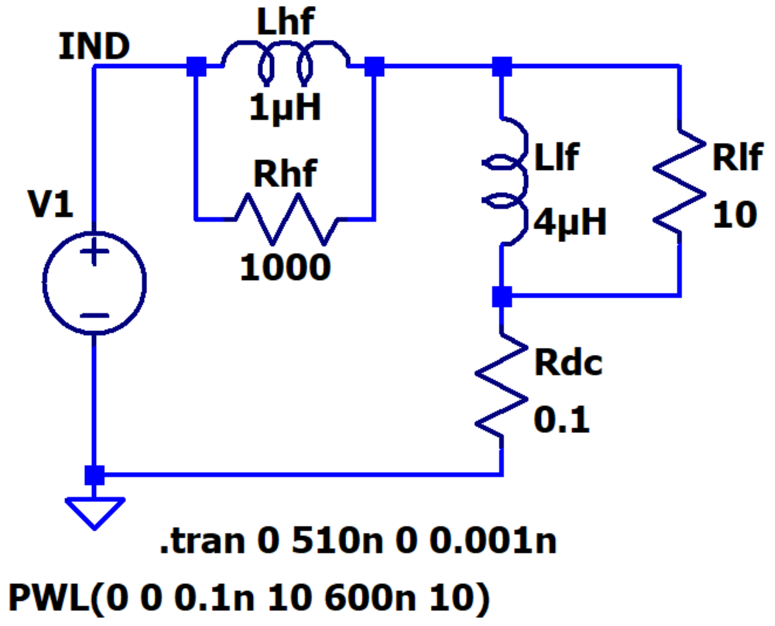
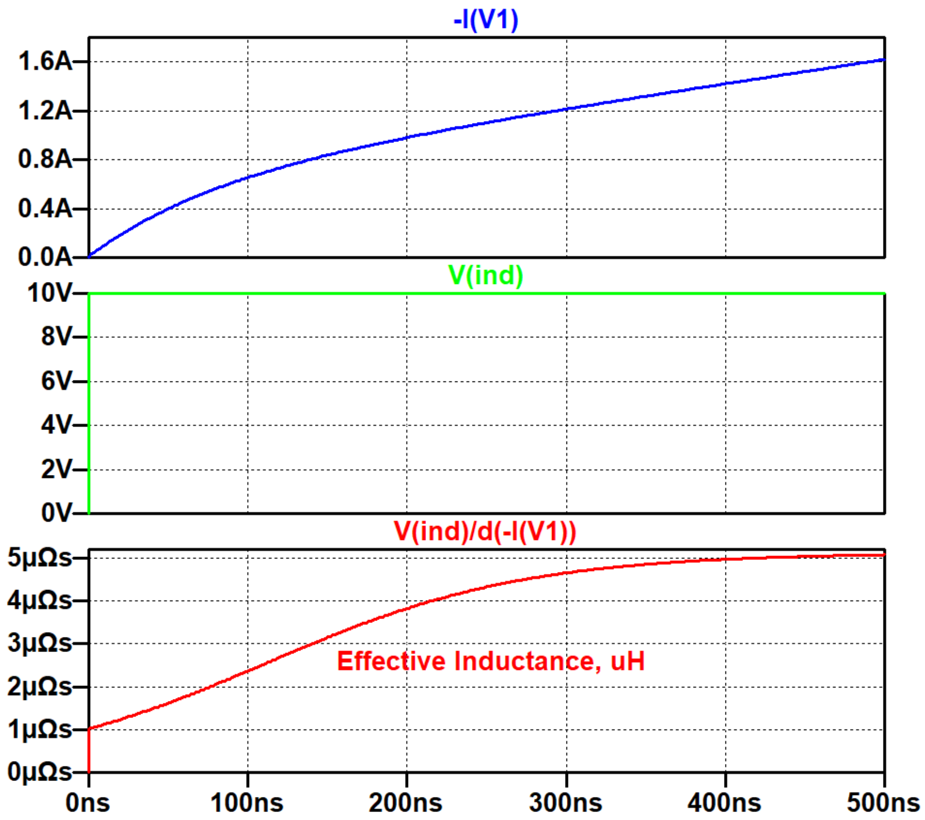
Pulse Measurement of Inductance



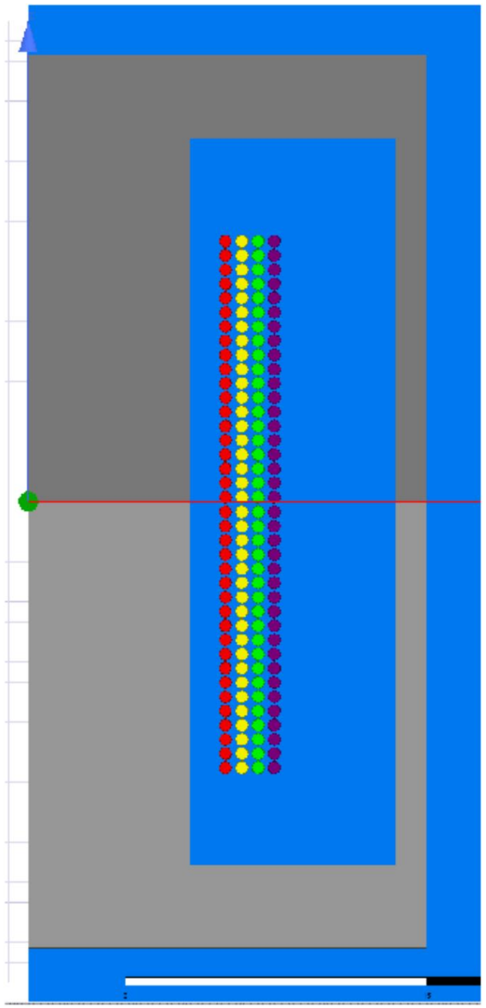


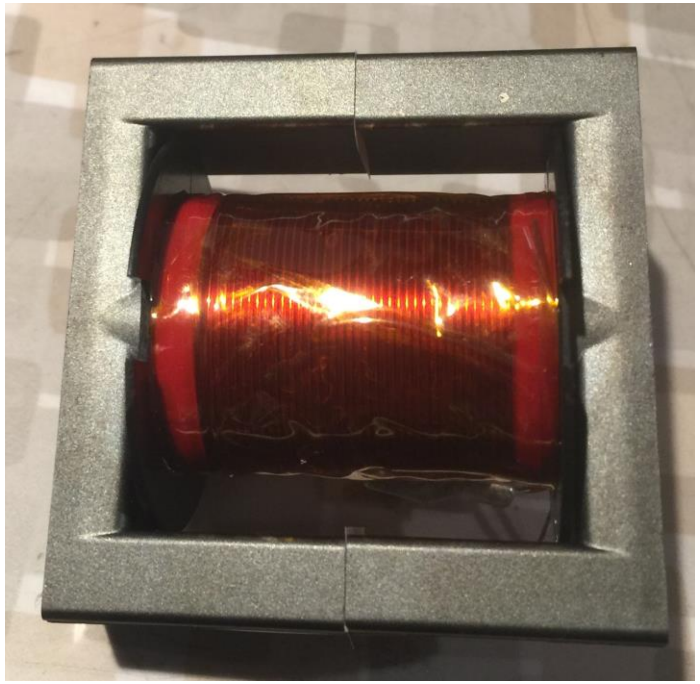
$$V_{ind} = L_{eff}\frac{di}{dt}$$

Effective Inductance $$L_{eff} = \frac{V_{ind}}{\frac{di}{dt}}$$

- At the beginning of the pulse, the effective inductance is equal to Lhf
- At the end of the pulse, the effective inductance is about equal to Lhf + Llf

Maxwell 2D Transformer Model





All wires 22 AWG

All windings 38 turns

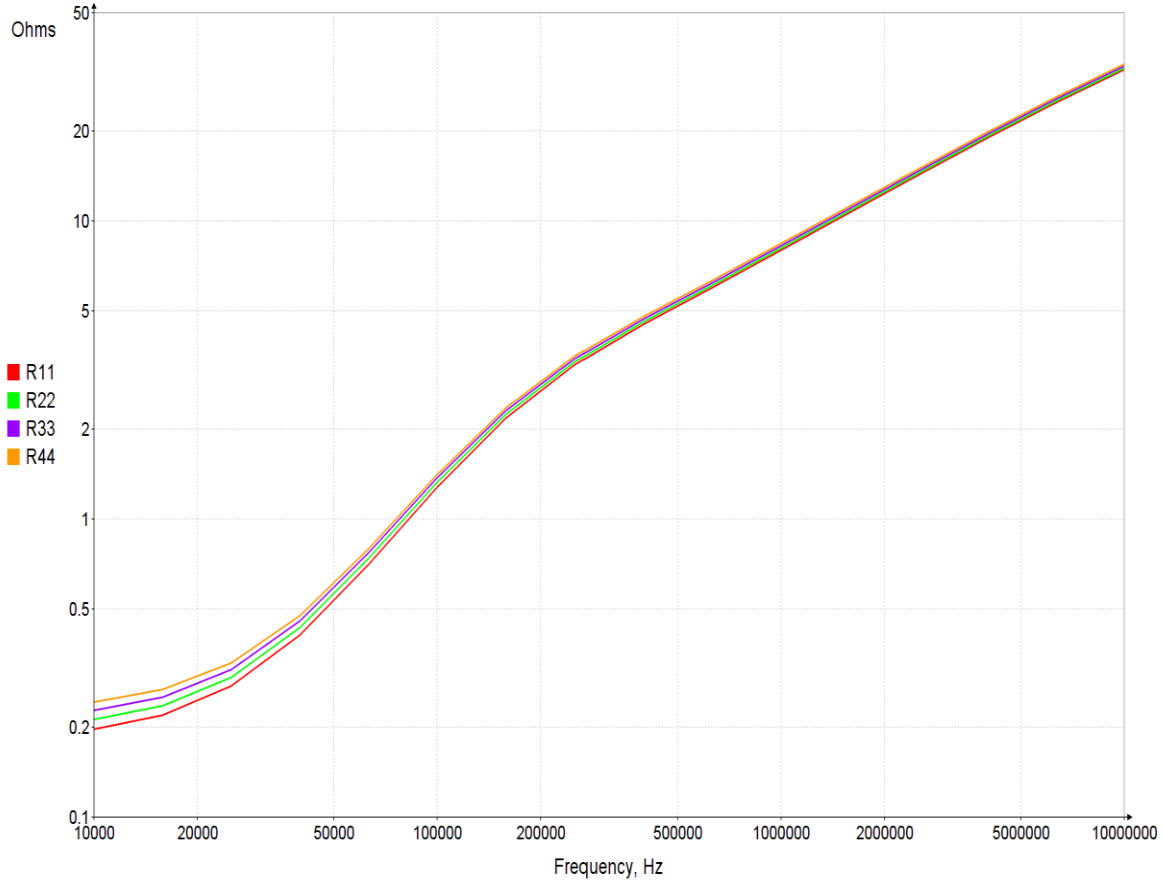
2 layers 2 mil tape between layers, 0.1mm

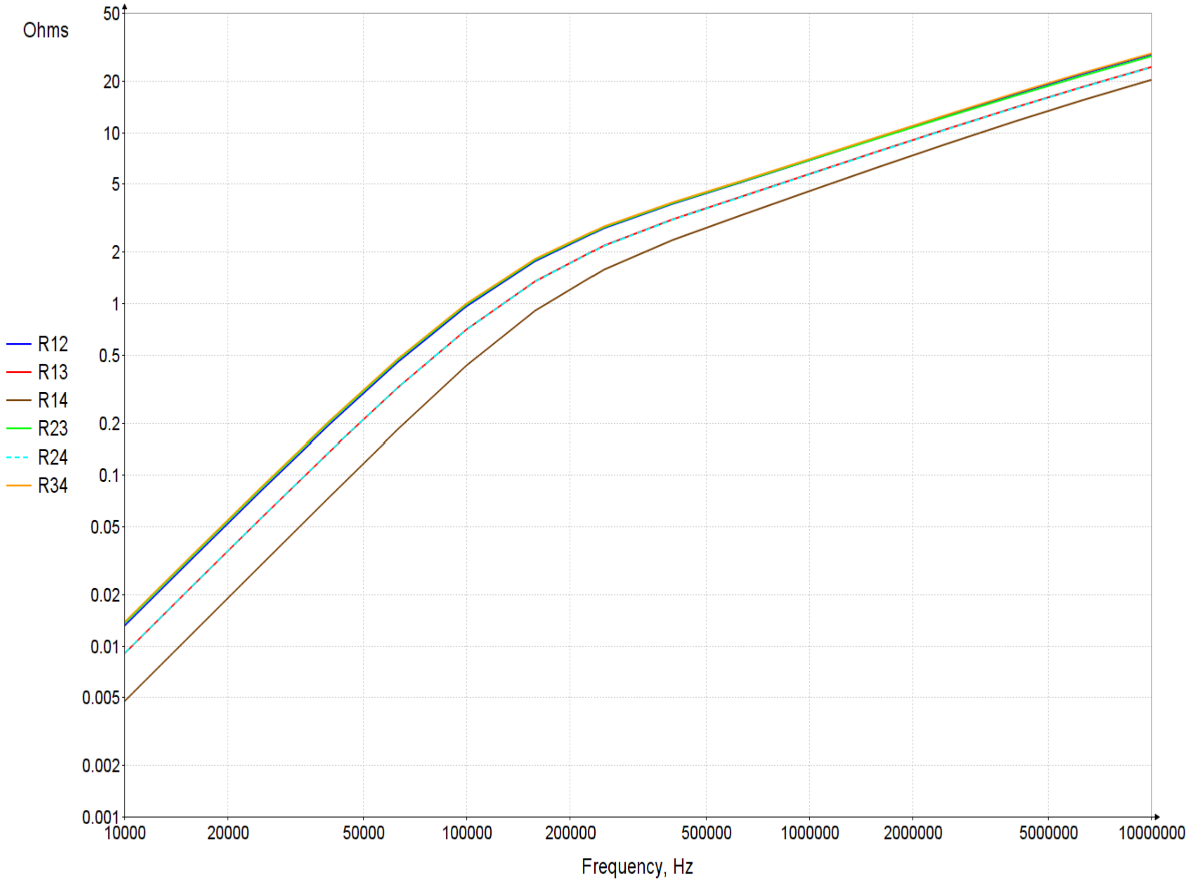
Core: ETD49/25/16-3C97

Gap: 3 mil spacer

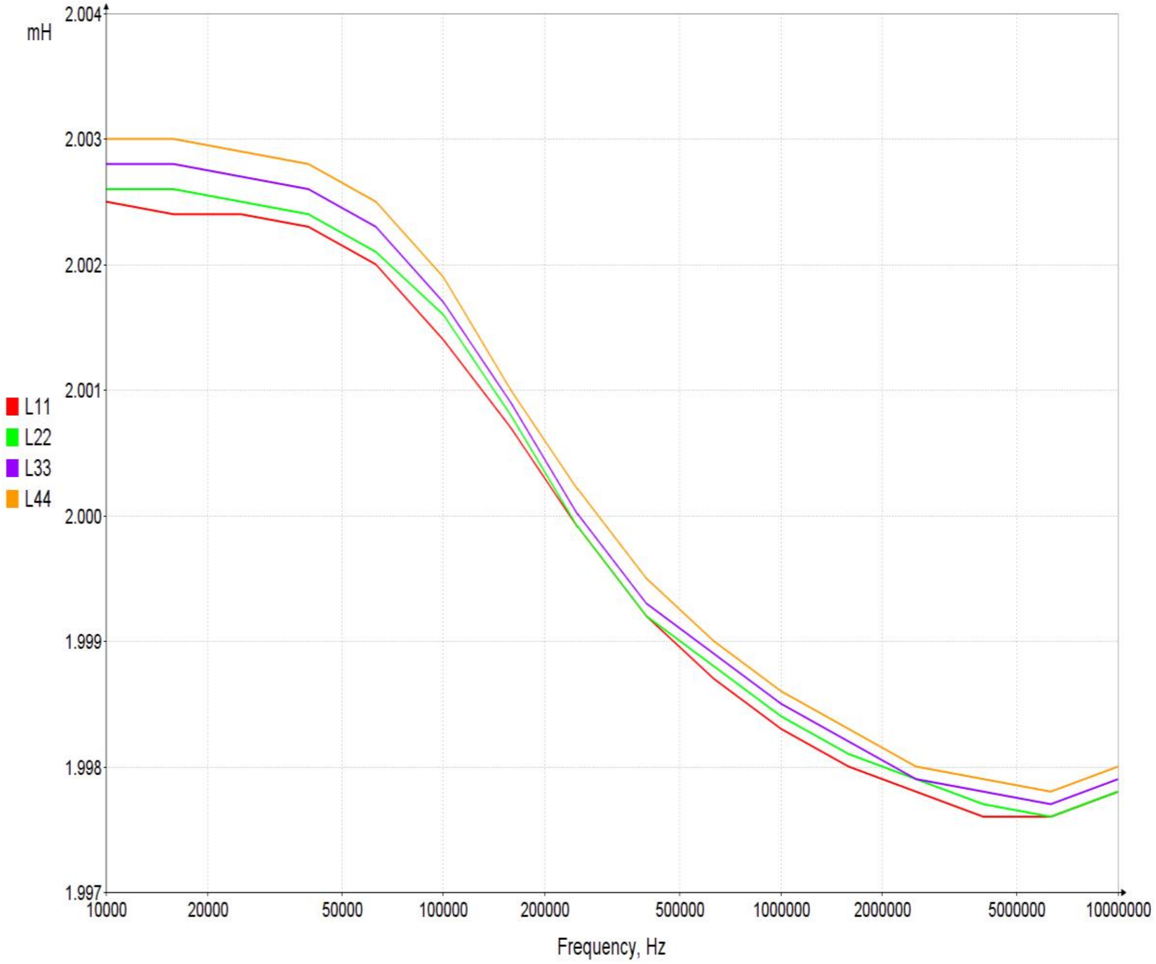
Bobbin: TDK B66368B1020T001

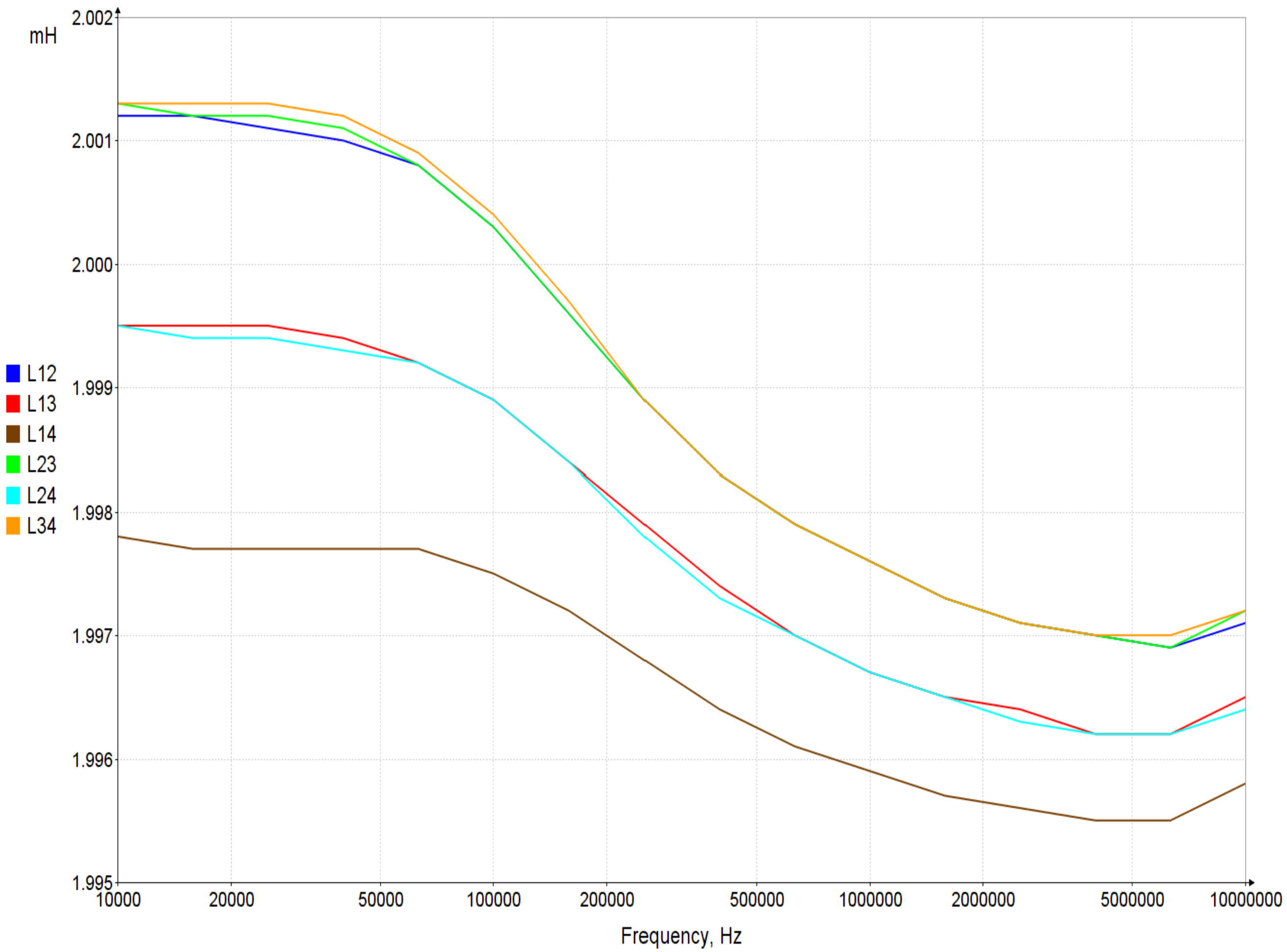
FEA Self and Mutual Resistances



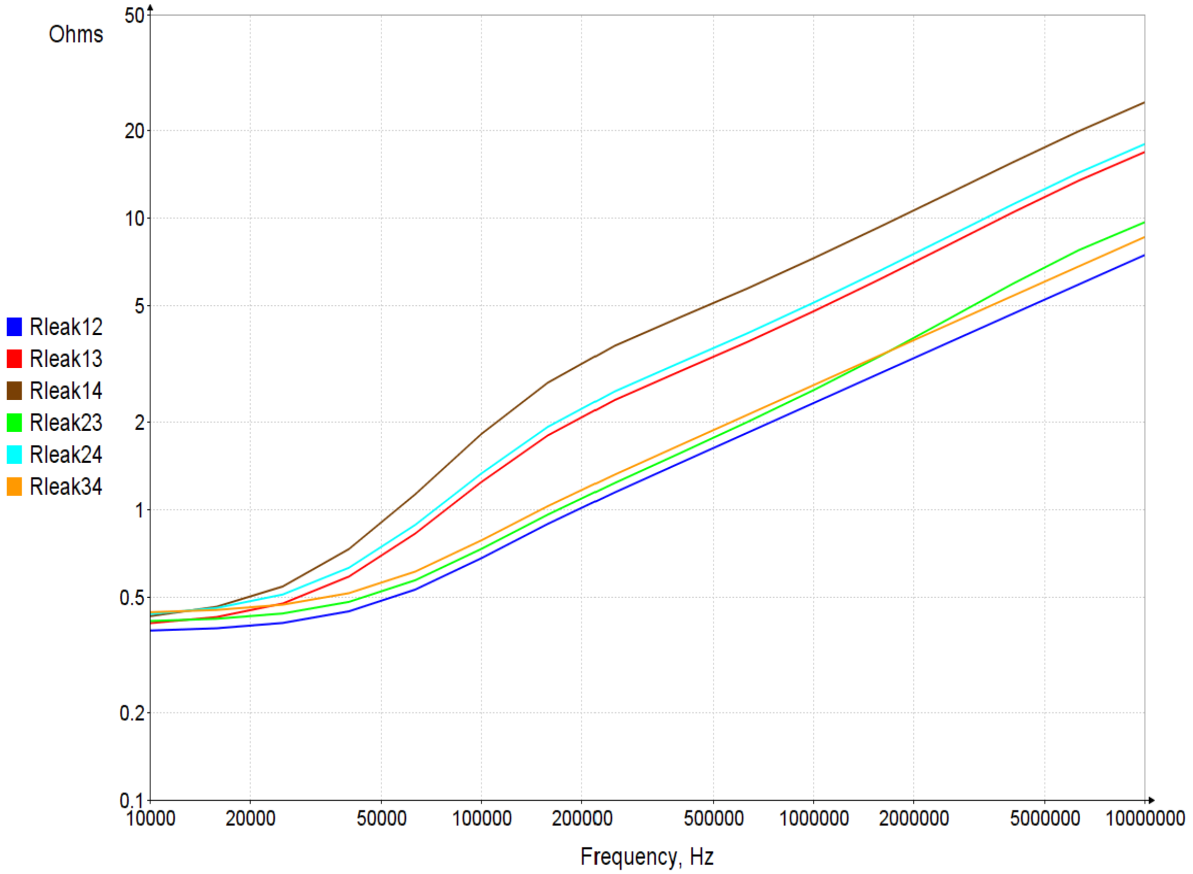


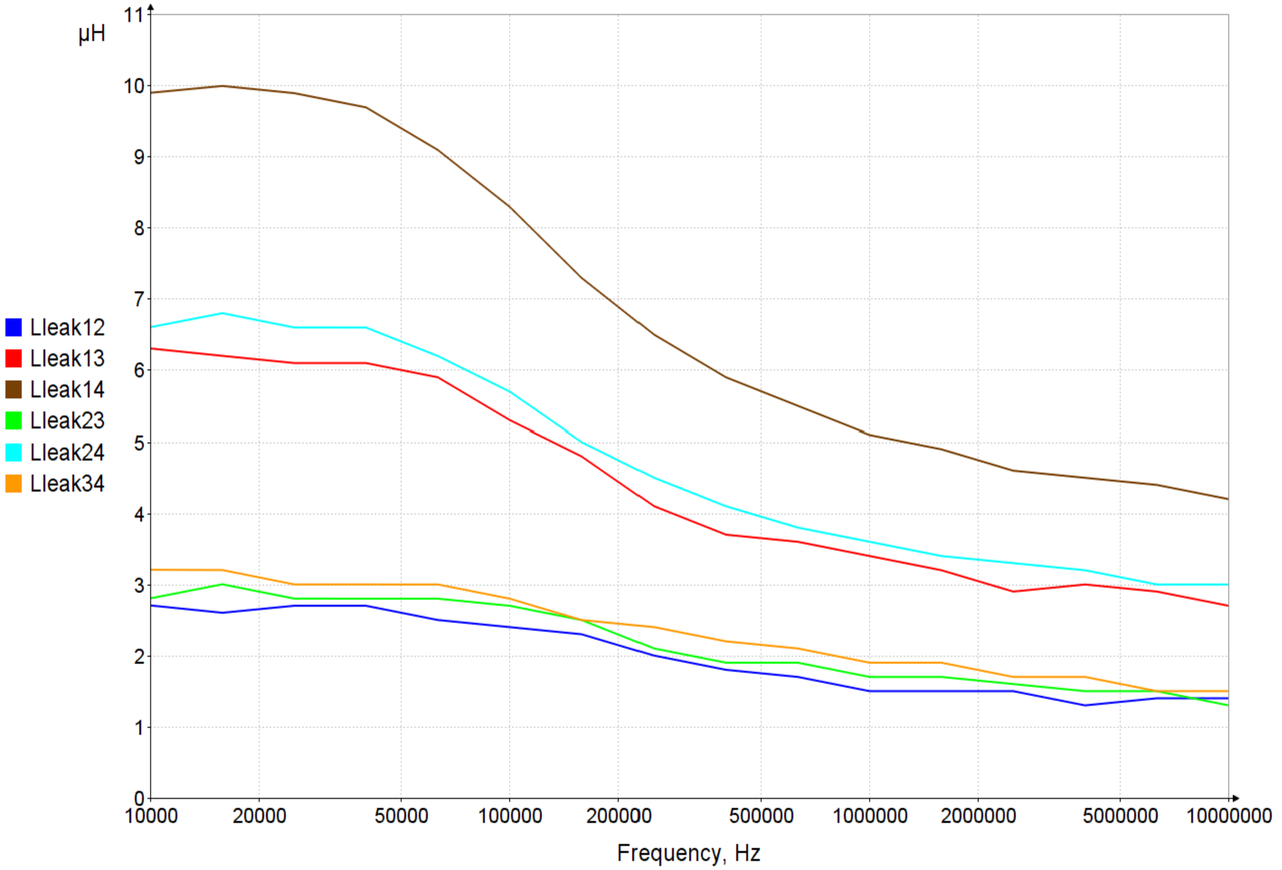
FEA Self and Mutual Inductances





FEA Leakage Resistances and Inductances





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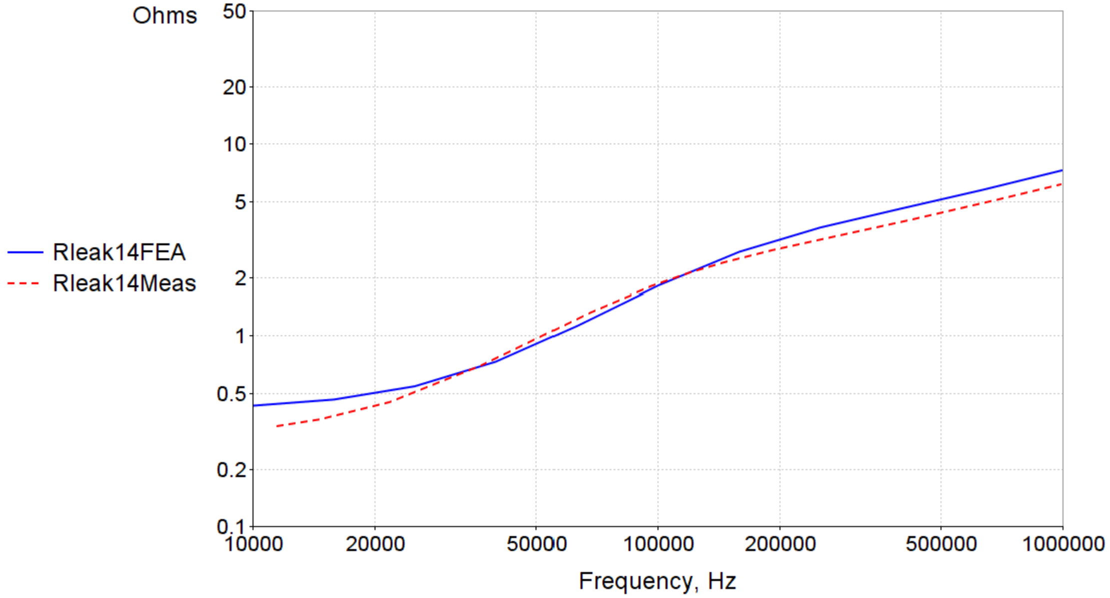
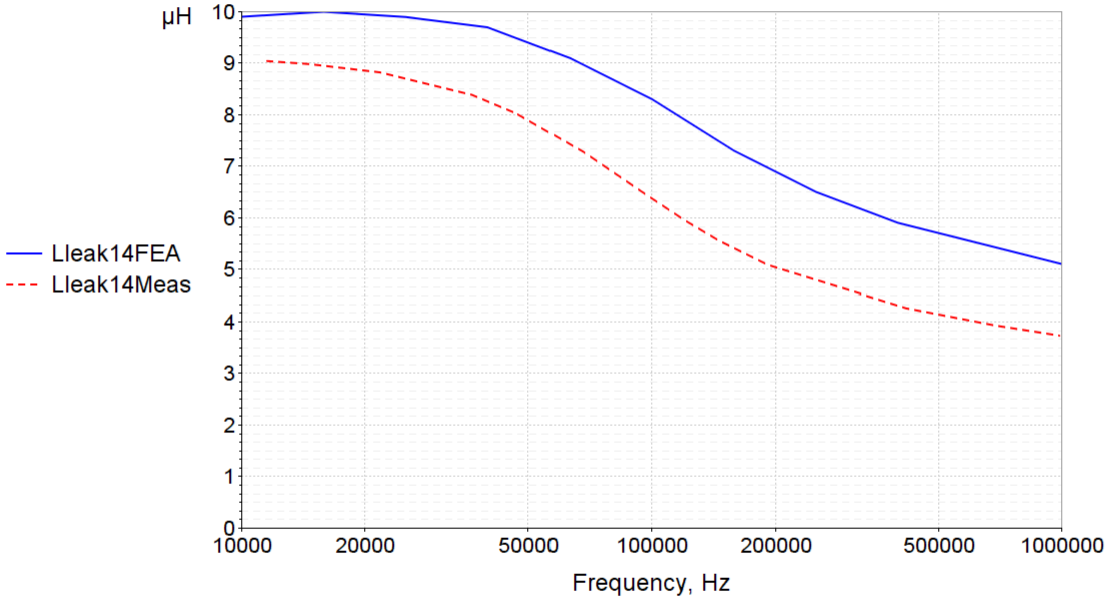
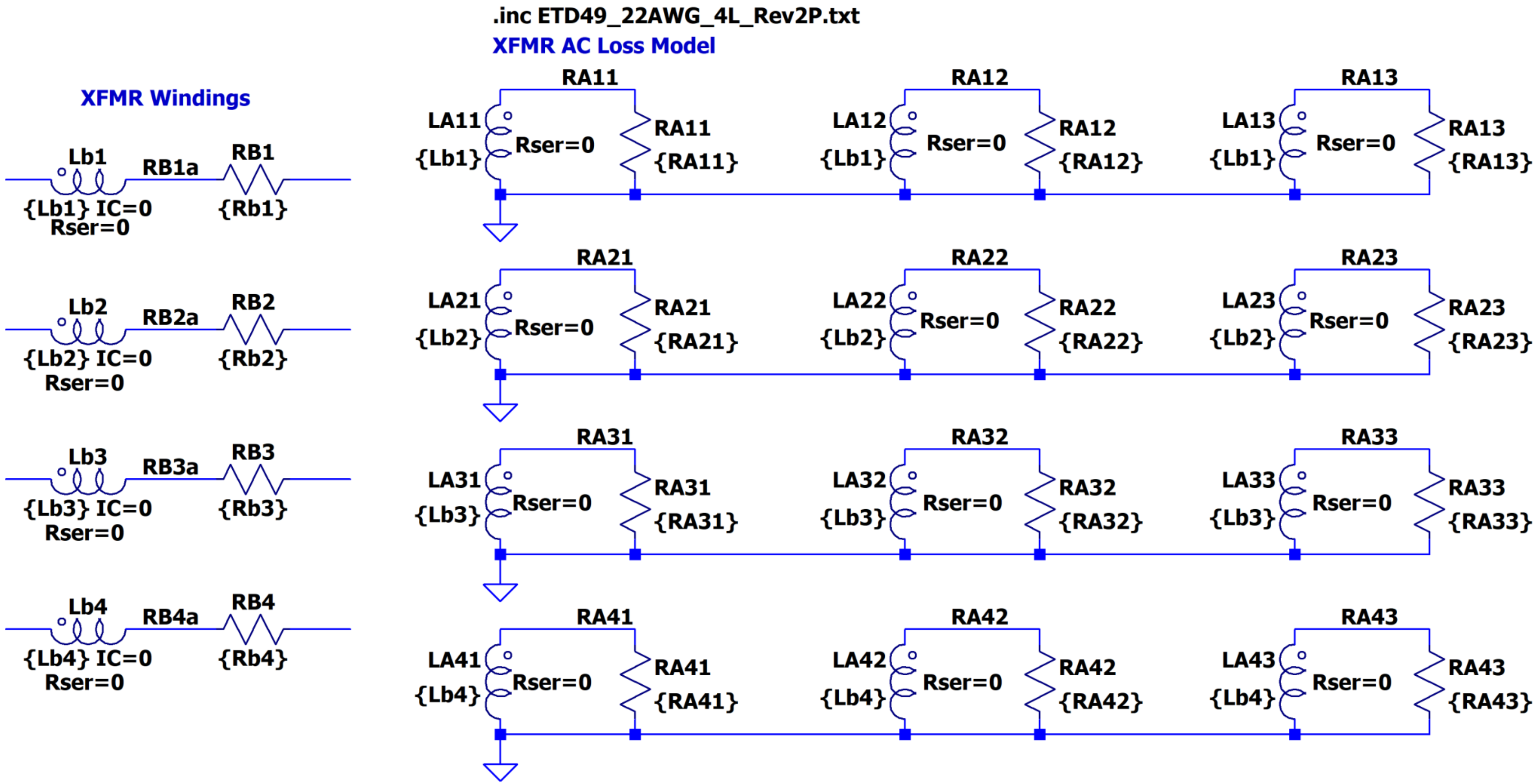
FEA and Measured Leakage Resistances and Inductances

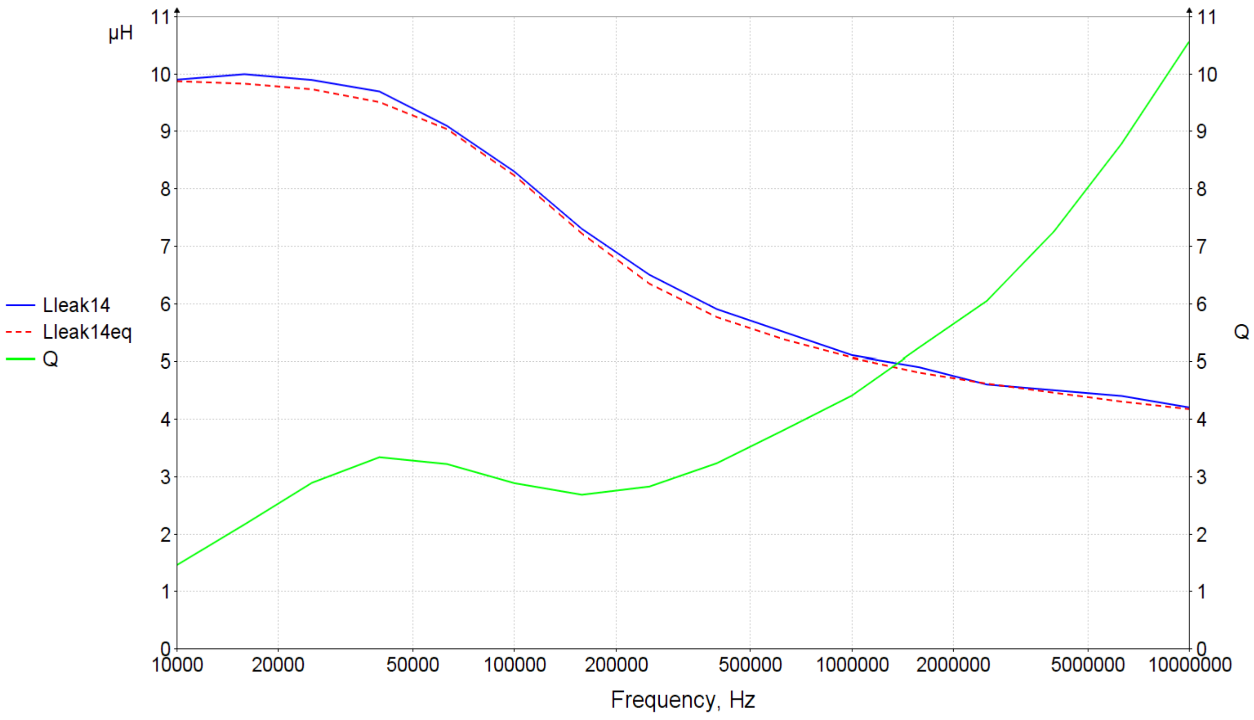


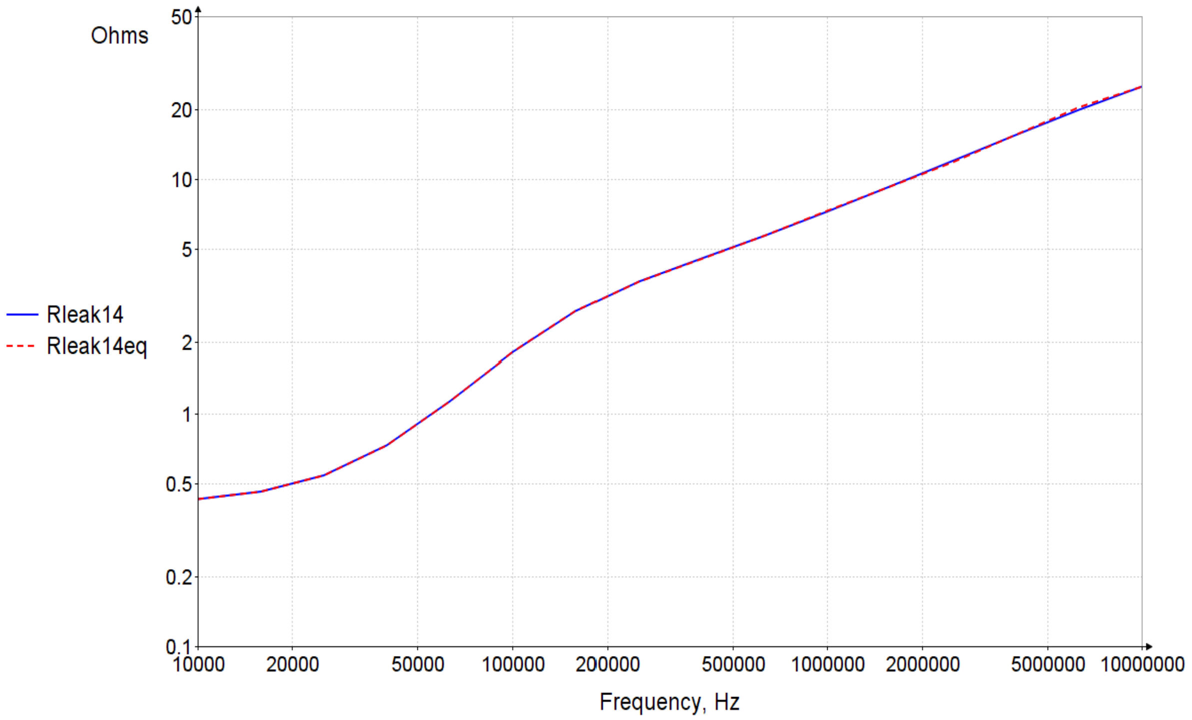


SPICE Mutual Impedance Transformer Model



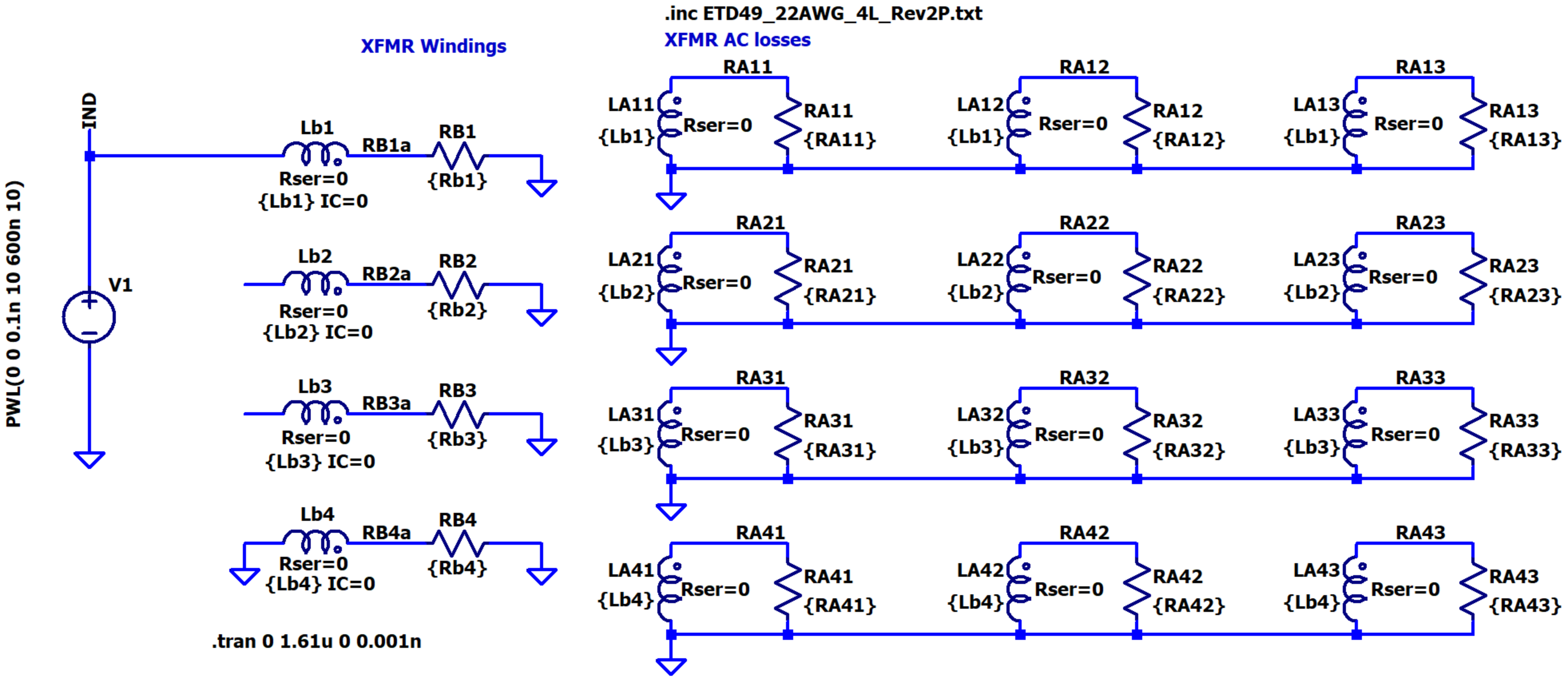
Equivalent Circuit Model Closely Matches FEA results



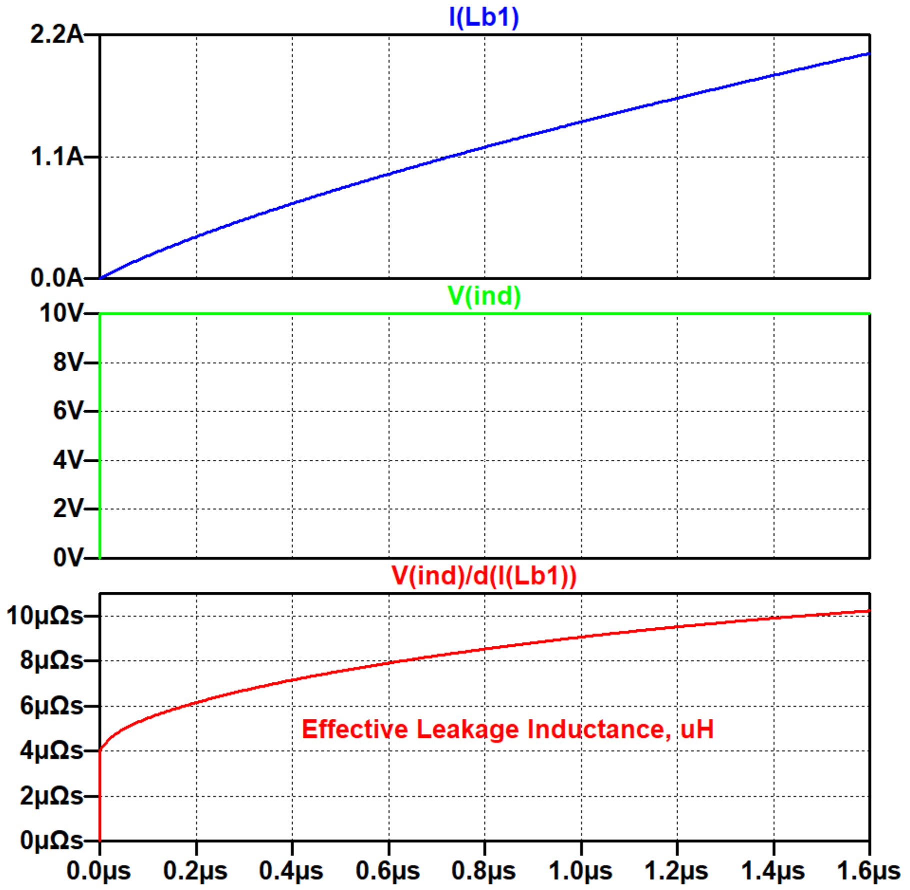


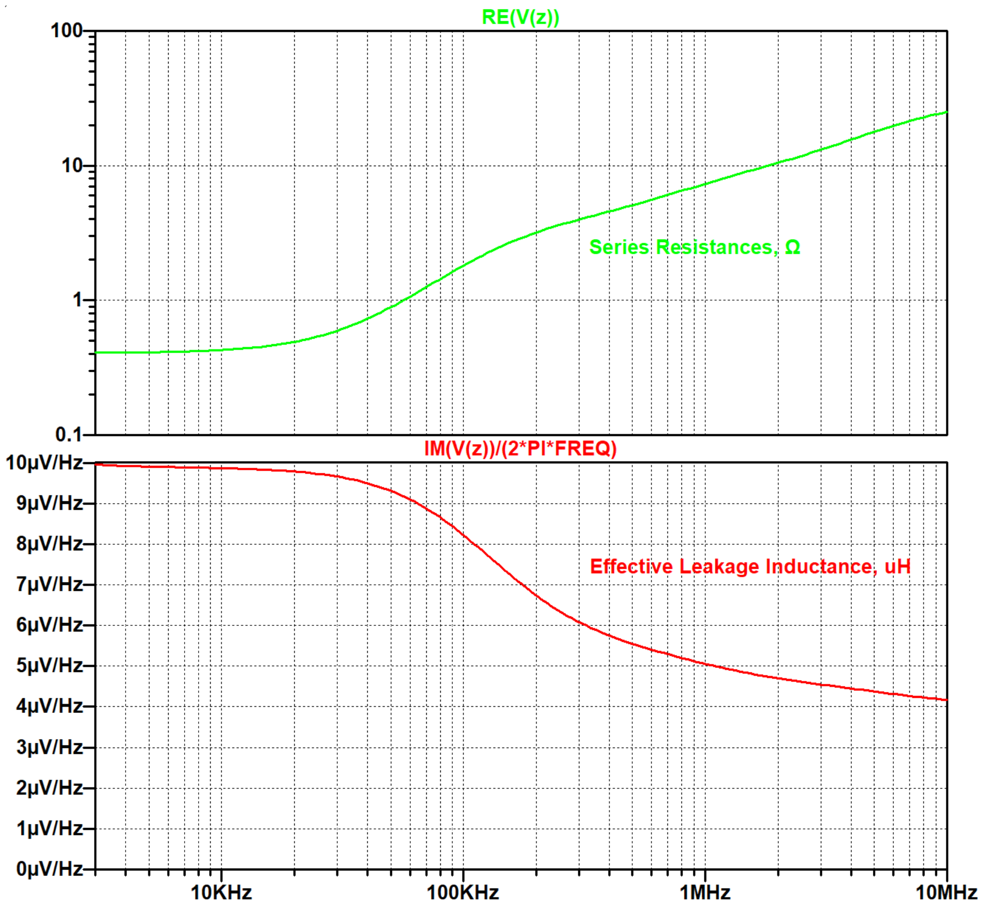
- The highest leakage impedances occur when measuring winding 1 with winding 4 shorted because those windings have the greatest separation
- The equivalent circuit model results closely match the FEA data

Leakage Inductance Pulse Test



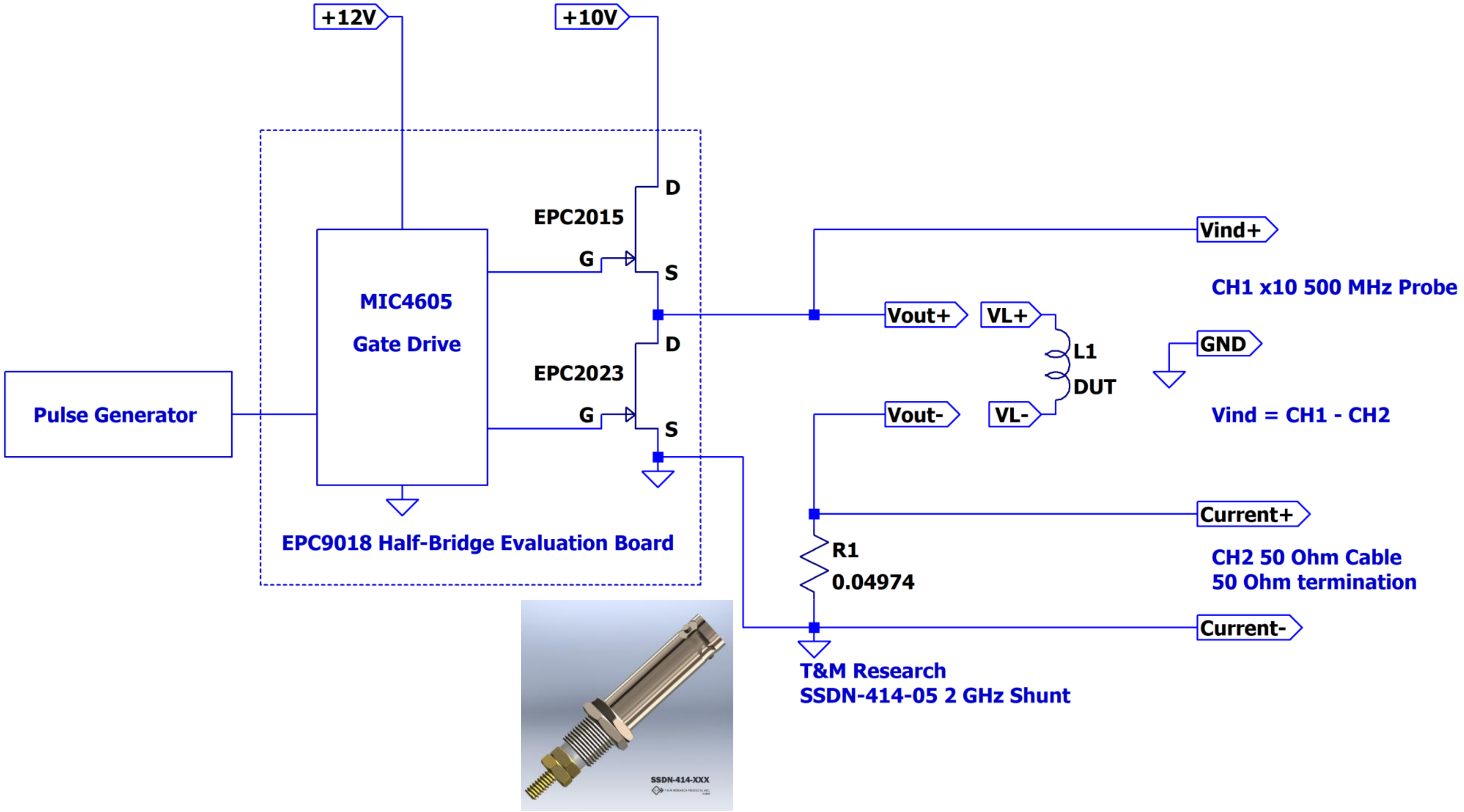
Leakage Inductance Simulations



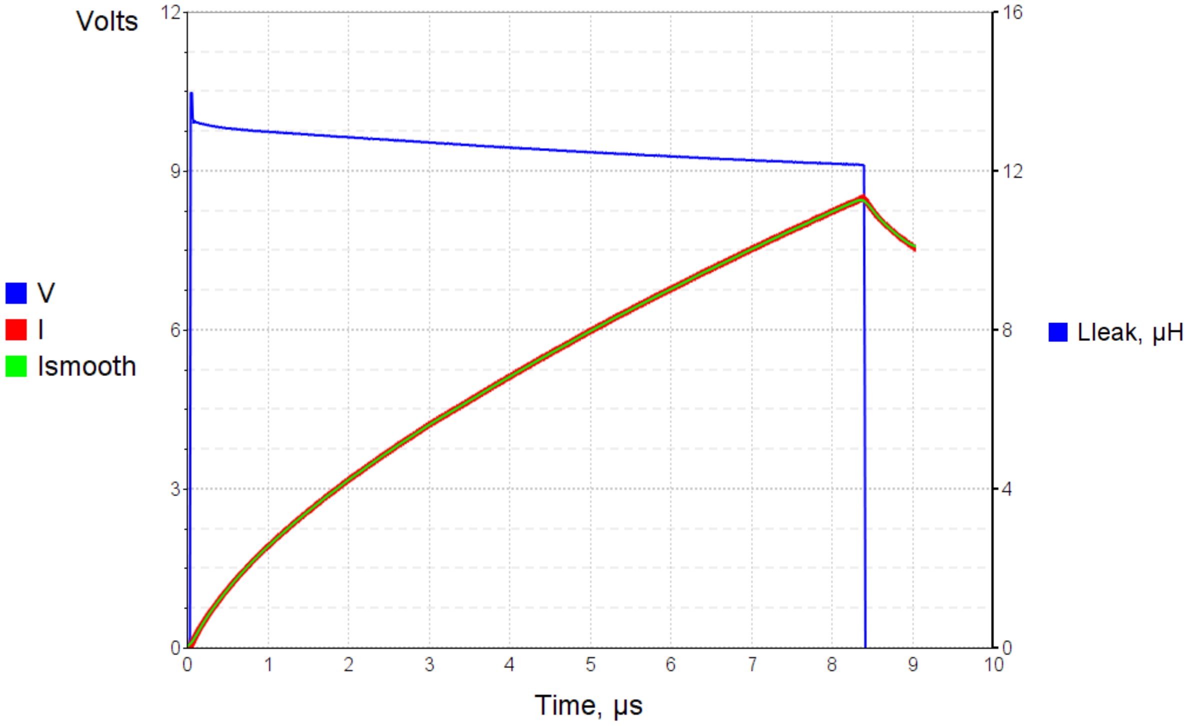


- The range for the effective leakage inductance in the time domain is close to the range in the frequency domain

Pulsing Test Fixture

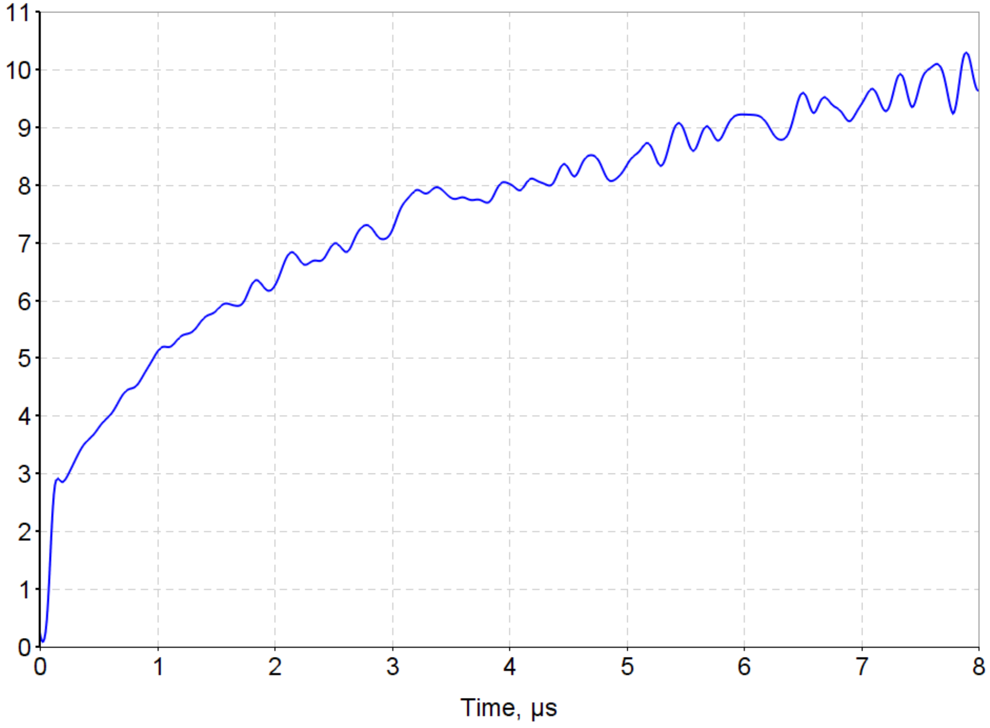


Leakage Inductance From Pulsed Waveforms



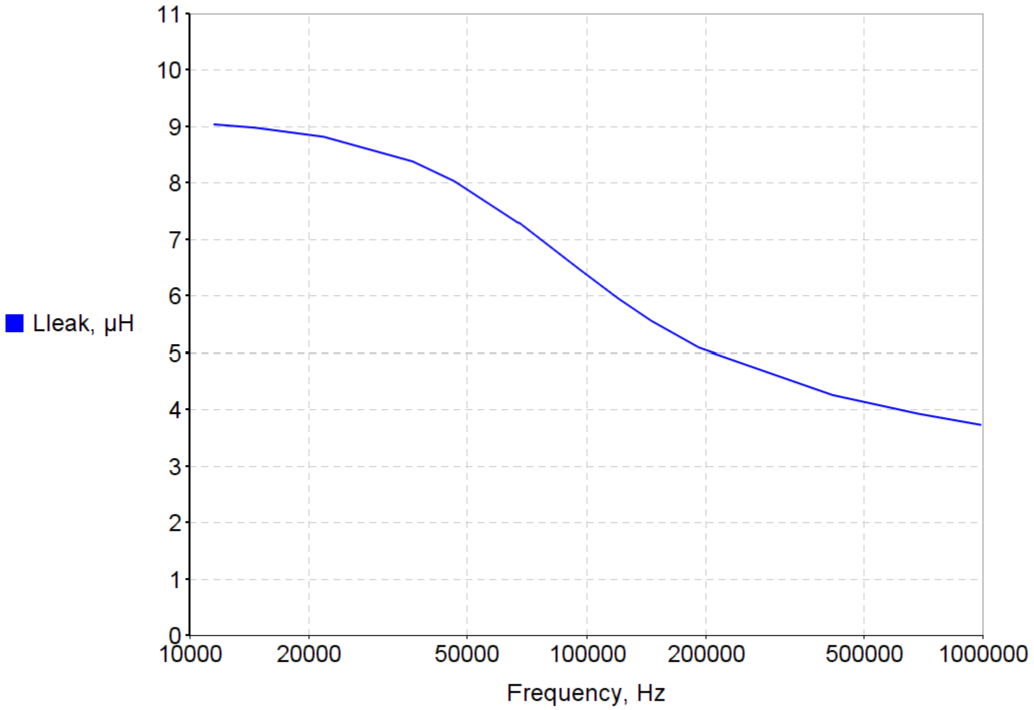
Oscilloscope current data smoothed in Mathcad

Oscilloscope data processed in Mathcad

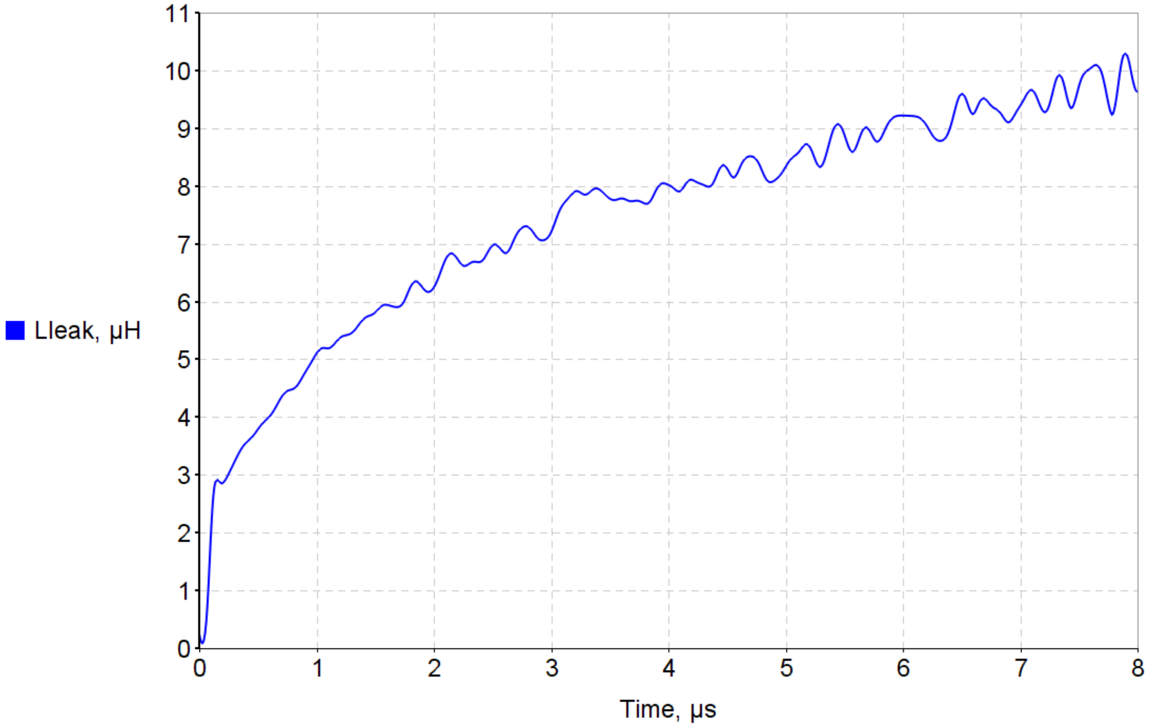


Effective Inductance $L_{eff} = \dfrac{V_{ind}}{\dfrac{di}{dt}}$

Measured and Extracted Leakage Inductance

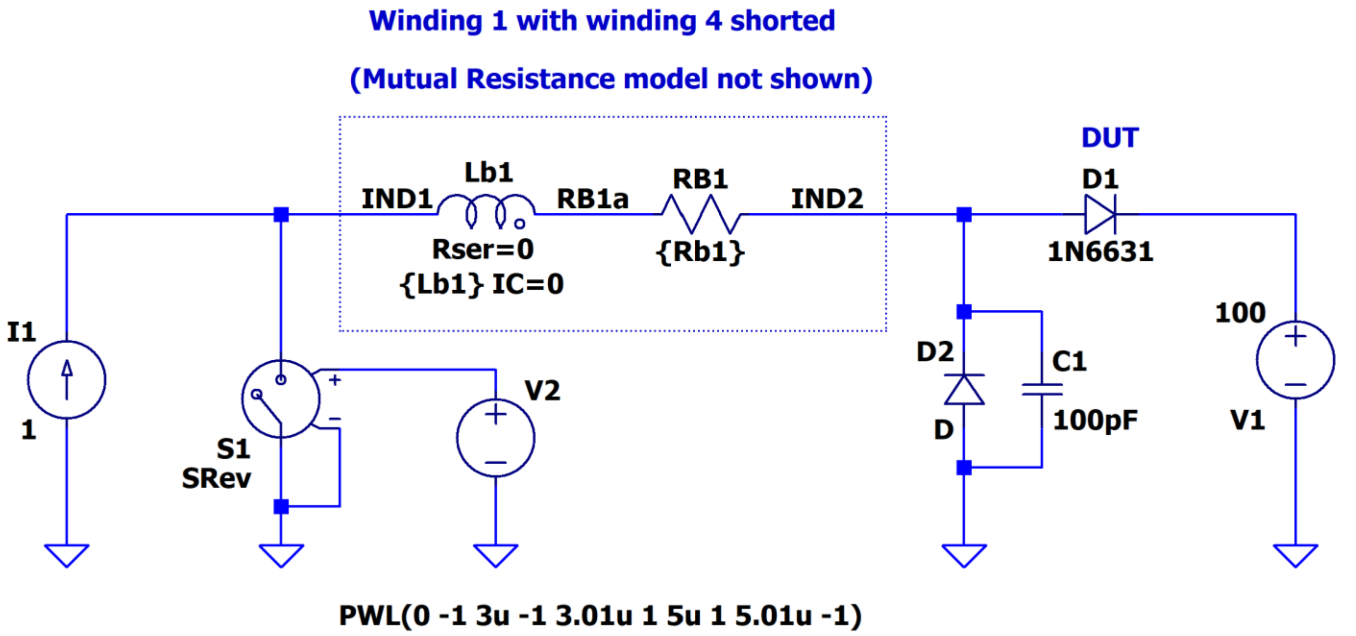


Measured leakage Inductance at winding 1 with winding 4 shorted



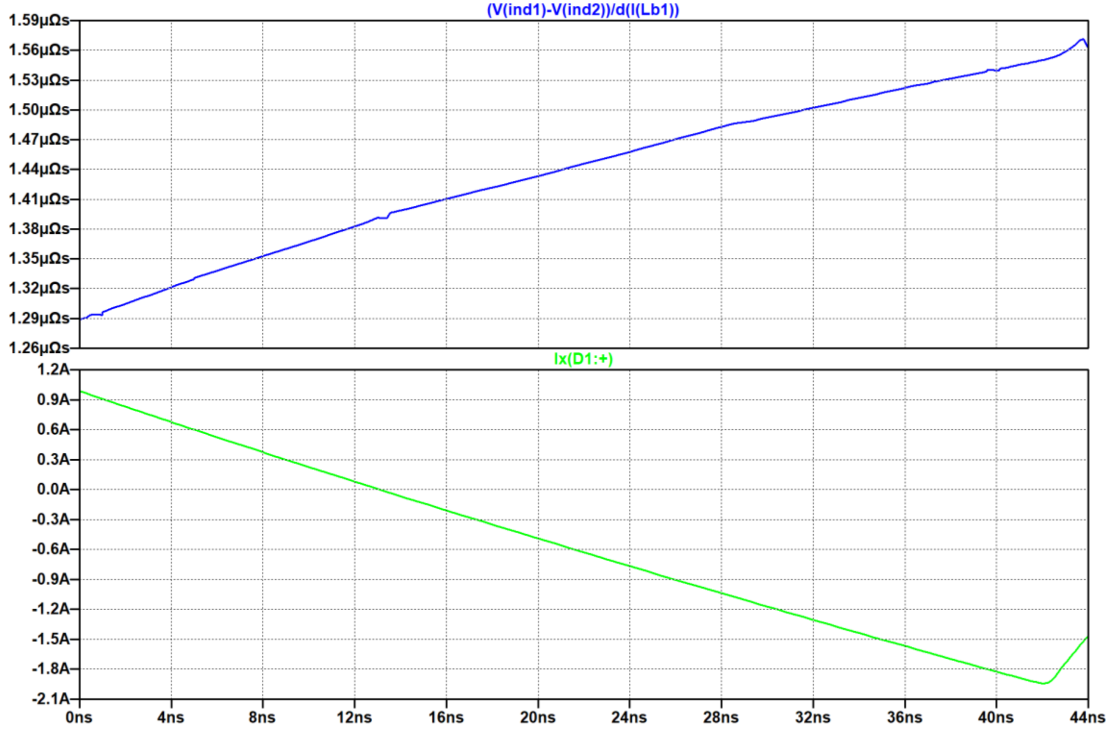
Leakage Inductance extracted from pulsed waveforms at winding 1 with winding 4 shorted

Diode Reverse Recovery Test Circuit

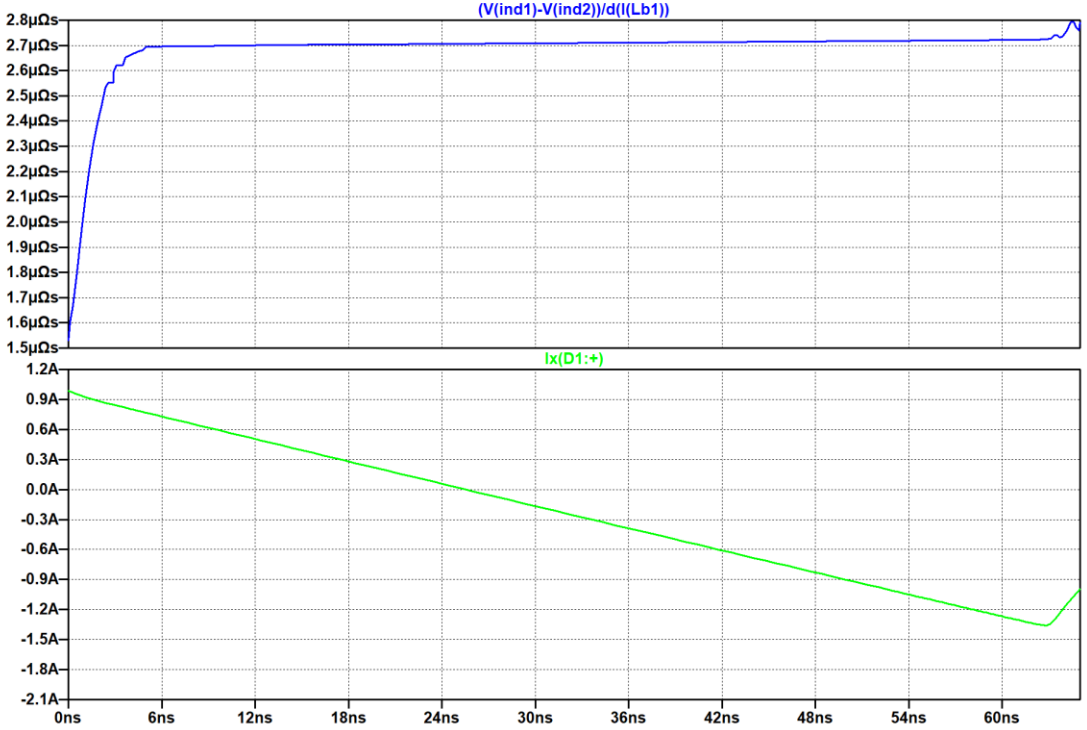



```
.SUBCKT 1N6631 anode cathode
.param IS=300n N=1.5 Tau=60n Y0=1 alpha=5MEG va=0.1
Dpn anode N001 Dj
.MODEL Dj D( IS {IS} N {N} )
Cdiff N002 diff 1
Vdiff diff 0 0V
Ediff  N003 0 Value {I(Vdiff)}
Hcsense N005 0 VHcsense 1
VHcsense N001 N004 0V
G_charge_current anode N004 N003 0 1
E_charge_calculator N002 0 VALUE{Tau*(V(N005)-va*V(N003))}
G_base_region_current N004 cathode VALUE {(V(N004) - V(cathode))*(Y0+alpha*V(N002)) }
.ENDS
```

Diode Reverse Recovery Comparison



Mutual Resistance Model Enabled



Mutual Resistance Model Disabled

- The mutual resistance model reduces the effective inductance and increases the peak reverse recovery current

Conclusions

- Leakage inductance for closely-coupled winding pairs decreases with frequency
- The inductance decrease is due to skin and proximity effects
- The effective leakage inductance for pulsed waveforms can be determined by dividing the applied voltage by the time derivative of the current
- Derivatives of measured data are typically noisy and need smoothing
- The effective leakage inductance for short pulses is less than for longer pulses
- The currents produced by the reverse recovery of fast diodes connected to transformer outputs depend on the high-frequency leakage inductance values