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Time-Domain K-Factor Computation Methods

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Abstract One method of rating the capability of transformers to handle harmonic currents requires the calculation of a coefficient related to eddy current losses called the K-factor. The standard method of calculating K-factors requires measurement of the magnitudes of the fundamental and harmonic components of current waveforms. Taking these measurements typically requires the use of expensive equipment. This paper describes relatively simple methods for computing K-factors that do not require the measurement of harmonics. The results of computer simulations are presented. The importance of appropriately limiting the number of harmonics included in K-factor computations is demonstrated.

1. INTRODUCTION

UL standards 1561 and 1562 introduced a term called the K-factor for rating transformers based on their capability to handle load currents with significant harmonic content. This method is based on the ANSI/IEEE C57.110-1986 standard, *Recommended Practice for Establishing Transformer Capability When Supplying Nonsinusoidal Load Currents.* In order to compute the K-factor formula, the individual harmonic components of the load current under consideration must be computed first. Time-domain methods of computing the K-factor do not require measurement or computation of individual harmonic currents. These methods have been developed as potentially lower cost alternatives to the more computationally intensive frequency-domain methods.

Two digital time-domain methods of K-factor computation are discussed by S. Wang and M. Devaney in [1]. This paper presents an additional digital method and also describes an analog computation method. In addition to describing attractive methods of performing K-factor computations, this paper uses the underlying principle of the time-domain calculations to show why the number of harmonics included in K-factor computations must be appropriately limited to avoid over specifying transformer requirements.

The K-factor is an estimate of the ratio of : (a) the heating in a transformer due to winding eddy currents when it is loaded with a given nonsinusoidal current to (b) the winding eddy-current heating caused by a sinusoidal current at the rated line frequency which has the same RMS value as the nonsinusoidal current. Suppose, for example, that the current in a transformer winding is 100 A, and that this current has a K-factor of 10. The eddy current losses in that winding will be approximately 10 times what they would be for a 100 A sinusoidal current at the rated line frequency. Transformers with K ratings are constructed so that their winding eddy current losses are very low for sinusoidal currents at the rated line frequency. This allows them to have acceptable losses when they are fully loaded with non-sinusoidal currents that have a K-factor less than or equal to the K rating of the transformer.

Although the K-factor formula was defined for transformer currents, K-factors of individual load currents are sometimes computed. This practice can be misleading because, in general, K-factors measured at transformers are significantly lower than the relatively high K-factors commonly measured at the input of individual electronic devices [2]. The reduction is primarily due to power system impedances and the essentially random phase angles of the harmonic currents produced by various loads. Another misleading practice is computing the K-factor of neutral currents. Nonlinear loads can cause excessive neutral currents, but the K-factor of neutral currents is not related to the AC losses of either neutral conductors or transformers.

The AC loss in a transformer winding is mainly due to the sum of the I^2R losses produced by the fundamental and harmonic components of the current, recognizing that for each component, R depends on the frequency of that component. Circulating currents between parallel conductors also produce AC losses, but this effect is not considered in K-factor calculations. For lower-order harmonics, the frequency dependence of the winding resistance is primarily due to the proximity effect, a phenomenon which occurs in coils because the magnetic field surrounding each conductor in a coil depends on the fields produced by other conductors. The proximity effect produces greater losses than those predicted by the skin effect, which is dominant at higher frequencies.

The K-factor formula does not account for the eddy currents losses and other losses that occur in transformer cores. Core losses due to harmonics depend primarily on the voltage distortion across the transformer windings. The voltage distortion appearing across the windings of a transformer carrying harmonic currents depends on the impedance of the transformer, the impedance of the system feeding the transformer, and the voltage distortion of that system. Although K-rated transformers are usually constructed to withstand more voltage distortion than other transformers, this capability cannot be directly determined from K ratings.

The K-factor formula is based on the assumption that the winding eddy current loss produced by each harmonic component of a nonsinusoidal current is proportional to the square of the harmonic order as well as being proportional to the square of the magnitude of the harmonic component. Unfortunately, this assumption is often stated without specifying that it is only valid over a limited range of frequencies. This issue is addressed in section 2. The K-factor is often defined in terms of per-unit quantities. It can also be defined by the following equivalent formula:

$$K = \frac{\sum_{h=1}^{\infty} (hI_h)^2}{\sum_{h=1}^{\infty} (I_h)^2} \tag{1}$$

where I_h is the RMS value of the current component at harmonic order h.

2. IDEAL TIME DOMAIN CALCULATION OF THE K-FACTOR

Suppose *i(t)* is a periodic current having a fundamental frequency f_1 that is the same as the nominal frequency of the power system. This current can be expressed as the Fourier series:

$$i(t)=\sum_{h=1}^{\infty}\sqrt{2}I_h\sin(2\pi hf_1t+\theta_h) \tag{2}$$

where I_h and θ_h are, respectively, the RMS value and the phase of the current component at harmonic order h.

The essence of calculating the K-factor in the time domain consists of appropriately processing a signal representative of *i(t)* so as to: (a) form a signal proportional to the numerator of equation (1), (b) form another signal proportional to the denominator of equation (1), and (c) divide the numerator signal by the denominator signal.

First consider the denominator of equation (1). Although it is not readily apparent, this summation is equal to the mean square value of *i(t)*, $\overline{i^2}$. Taking the mean square of both sides of equation (2) results in:

$$\overline{i^2}=f_1\int_0^{\frac{1}{f_1}}\left[\sum_{h=1}^{\infty}\sqrt{2}I_h\sin(2\pi hf_1t+\theta_h)\right]^2dt \tag{3}$$

When the square of the summation in equation (3) is expanded, it results in an infinite summation of terms, each of which is the product of two sinusoidal terms. There are terms composed of the product of two identical sinusoids and terms composed the product of two sinusoids which have different values of *h*. After integration from 0 to $\frac{1}{f_1}$, which is an integral number of cycles for any harmonic, only the terms where both sinusoids have the same value of *h* are non-zero. This leads to the following simplification:

$$\overline{i^2}=f_1\sum_{h=1}^{\infty}2I_h^2\int_0^{\frac{1}{f_1}}\left[\sin(2\pi hf_1t+\theta_h)\right]^2dt \tag{4}$$

Integrating the terms in equation (4) and simplifying produces:

$$\overline{i^2}=\sum_{h=1}^{\infty}I_h^2 \tag{5}$$

Thus the denominator of the K-factor is simply $\overline{i^2}$, which can be directly computed by analog or digital methods. Alternatively, $\overline{i^2}$ can be found by squaring RMS value of *i(t)*.

To derive a formula for the numerator of the K-factor, consider the fact that each harmonic component is multiplied by its harmonic order. This suggests taking the derivative of *i(t)* in equation (2) with respect to time, and then dividing both sides by $2\pi f_1$:

$$\frac{1}{2\pi f_1}\frac{di}{dt} = \sum_{h=1}^{\infty} \sqrt{2} h I_h \cos(2\pi h f_1 t + \theta_h) \qquad (6)$$

Taking the mean square of both sides produces:

$$\overline{\left(\frac{1}{2\pi f_1}\frac{di}{dt}\right)^2} = f_1 \int_0^{\frac{1}{f_1}} \left[\sum_{h=1}^{\infty} \sqrt{2} h I_h \cos(2\pi h f_1 t + \theta_h)\right]^2 dt \qquad (7)$$

Following the procedure used to simplify equation (3) results in:

$$\overline{\left(\frac{1}{2\pi f_1}\frac{di}{dt}\right)^2} = f_1 \sum_{h=1}^{\infty} 2(hI_h)^2 \int_0^{\frac{1}{f_1}} [\cos(2\pi h f_1 t + \theta_h)]^2 dt \qquad (8)$$

Integration and simplification of equation (8) produces a time-domain expression for the numerator of equation (1):

$$\overline{\left(\frac{1}{2\pi f_1}\frac{di}{dt}\right)^2} = \sum_{h=1}^{\infty} (hI_h)^2 \qquad (9)$$

Substituting equations (5) and (9) into equation (1) produces an equivalent formula for K that is suitable for time-domain computation:

$$K = \frac{\overline{\left(\frac{1}{2\pi f_1}\frac{di}{dt}\right)^2}}{\overline{i^2}} \qquad (10)$$

The above formula provides an exact value for the K-factor of any continuous current waveform. Unlike the standard method that is based on a finite number of calculated or measured harmonics, equation (10) includes all of the harmonics present in *i(t)*. Although the K-factor had not yet been invented, the right side of equation (10) was applied by S. Crepaz in 1970 to predict eddy current losses in transformer windings of polyphase rectifier circuits [3]. In all cases the predicted losses exceeded the measured losses, which suggests limiting the number of harmonics used in K-factor calculations. A theoretical basis for this is discussed below in more detail.

Besides providing an alternate calculation method, equation (10) shows why currents with sharp steps or jumps have large K-factors. The derivative in the numerator becomes very large for fast-rising waveforms. In fact, the K-factor of discontinuous waveforms such as perfect square waves is infinite. Another way of demonstrating this is to consider the convergence of equation (1). The denominator is recognizable as the square of the RMS value of the current, so that series will obviously converge for any current with a finite RMS value. The numerator series will converge for all continuous waveforms, because, for continuous waveforms, I_h decreases at least as fast as $\frac{1}{h^2}$ when h is sufficiently large [4]. Therefore, $(hI_h)^2$ will eventually decrease at least as fast as $\frac{1}{h^2}$, thereby ensuring convergence.

For discontinuous waveforms, I_h will have terms that decrease in proportion to $\frac{1}{h}$. This implies that the numerator series will not converge since each term is multiplied by h. In fact, for an ideal square wave, all of the terms in the series are equal. This is why the K-factor measurements of current waveforms produced by some rectifier circuits are roughly proportional to the number of harmonics being measured until, at sufficiently high frequencies, the harmonics are limited by circuit inductances. As an extreme example, suppose one tried to use either of the standard or time-domain K-factor formulas to predict the heating caused by high-frequency emissions from a switching power supply. These formulas predict that 20 μA of current at 30 Mhz will produce as much heating as 10 A at 60 Hz.

The K-factor formulas overestimate the high-frequency losses in transformer windings because of the assumption that the eddy current losses are proportional to the square of the frequency for all frequencies. According to the formulas in [3], [5]-[8], for high enough frequencies, winding eddy current losses in transformers are asymptotically proportional to the square-root of the frequency instead of the square of the frequency. The geometry of the windings in a given transformer determines when the transition between the square and square-root regimes occurs.

Figure 1 shows a plot of eddy current losses normalized to 1 at 60 Hz for an aluminum winding in a typical 45 KVA 480-120 V transformer according to the method in [3]. In this example, the transition between the two regimes occurs at 2193 Hz, or about the 37th harmonic. For a copper winding of the same size as the one in this example, the transition occurs at 695 Hz, which is about the 12th harmonic. The fact that the winding eddy-current losses are not proportional to the square of the frequency beyond some point is recognized in footnote 3 of ANSI/IEEE C57.110-1986, which states that, due to the skin effect, the methods of caculating transformer capablities presented in that document are increasingly conservative beyond the 19th harmonic. In accordance with this, the maximum harmonic used in computing K-factors has typically ranged from the 17th to the 33rd, even though it is possible to measure higher harmonics. Recently-developed K-factor computing instruments that have an upper limit of the 50th harmonic may lead to using transformers with unnecessarily high K-ratings if the current waveform under consideration has measurable levels of higher-order harmonics at frequencies above the transition frequency for the type of transformer being considered.

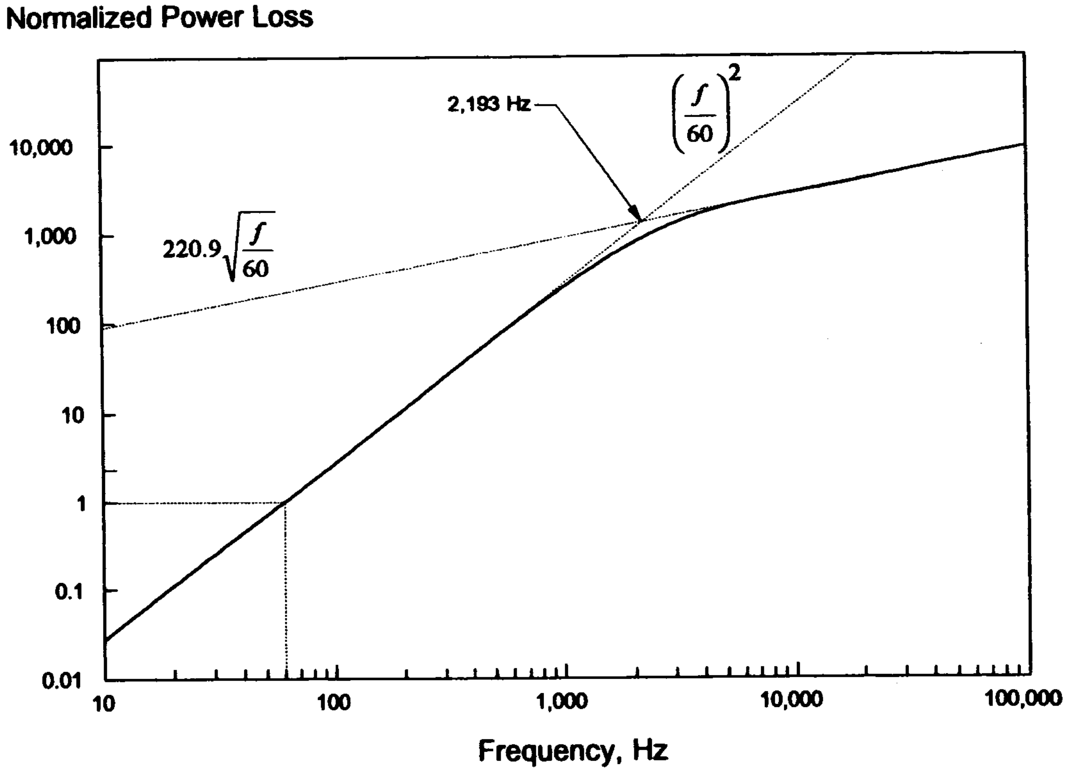

Figure 1. Theoretical Eddy Current Losses for a Typical Transformer Winding.

3. BANDLIMITED CALCULATION OF K-FACTORS

Since the maximum harmonic used in calculating K-factors must be limited to some finite value, it is useful to define a K-factor approximation, K_N, which includes the first N harmonics:

$$K_N = \frac{\sum_{h=1}^{N} (hI_h)^2}{\sum_{h=1}^{N} (I_h)^2} \tag{11}$$

When K-factors are computed in the frequency domain, the summations are simply carried out to the desired maximum harmonic, N. In the time domain, however, the maximum frequency must be defined with a low-pass filter. Let $G(hf_1)$ represent the gain of such a low-pass filter at the frequency hf_1. Another K-factor approximation, K_{Nf}, which includes the effect of the low-pass filter can be defined as:

$$K_{Nf} = \frac{\sum_{h=1}^{\infty} \left(G(hf_1)hI_h\right)^2}{\sum_{h=1}^{\infty} \left(G(hf_1)I_h\right)^2} \tag{12}$$

If $i_G(t)$ is used to denote the filtered *i(t)* signal, then K_{Nf} can be written in the time domain as:

$$K_{Nf} = \frac{\overline{\left(\frac{1}{2\pi f_1}\frac{d}{dt}i_G(t)\right)^2}}{\overline{\left(i_G(t)\right)^2}} \tag{13}$$

The cutoff frequency of the low-pass filter should be placed just above the highest harmonic of interest. Intuitively, filters which have a maximally flat passband should be better for this application than equiripple types.

The question may arise as to whether or not the phase distortion of the low-pass filter will affect the mean-square value of the current derivative. Examination of equation (8) reveals that the value of θ_h does not affect the result of the integration, so phase distortion is not a problem. By a similar argument, phase distortion will not affect the mean-square value of the current either.

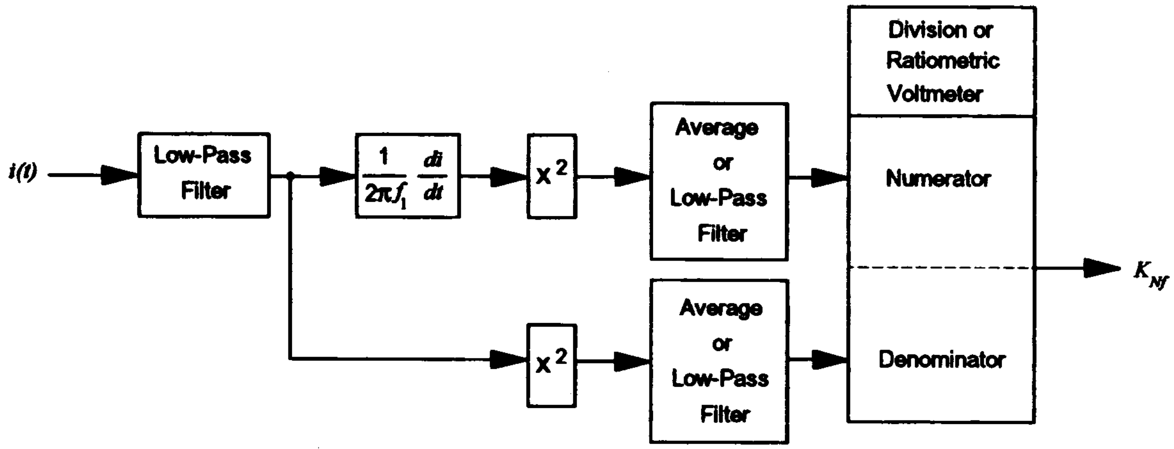

Figure 2. Time-Domain K-Factor Computation Method.

The block diagram in Figure 2 represents a method for calculating K-factors that is suitable for either analog or digital implementation. If analog computation is used, then the division operation can be accomplished with a ratiometric digital voltmeter, provided that the numerator is reduced by an appropriate scale factor. If a digital method is used, then the input filter also serves as an anti-aliasing filter.

Providing a way to compute the derivative is the most difficult part of designing a time-domain K-factor calculation scheme. In digital implementations, simply taking the difference between successive samples approximates the derivative only for frequencies that are far below the Nyquist frequency. An algorithm for calculating K-factors that uses second-order sampling is described in [1]. An accurate differentiator can also be implemented with a digital FIR filter. One method of designing FIR filter differentiators is described below in section 5.

Analog differentiators may lack sufficient dynamic range, particularly for waveforms with sudden jumps or impulses. A solution to this problem is found in the low-pass input filter, which smoothes the current waveform before it is differentiated by reducing high-order harmonics. It also appears that the phase distortion of low-pass filters tends to reduce the peak amplitude of the differentiated current signal by reducing the slope of rapidly changing current signal waveforms. This effect can be seen by observing the rising slope of the step responses of filters that have the same cutoff frequency, but which have different levels of phase distortion near the cutoff point.

4. EXAMPLE ANALOG COMPUTATION OF THE K-FACTOR

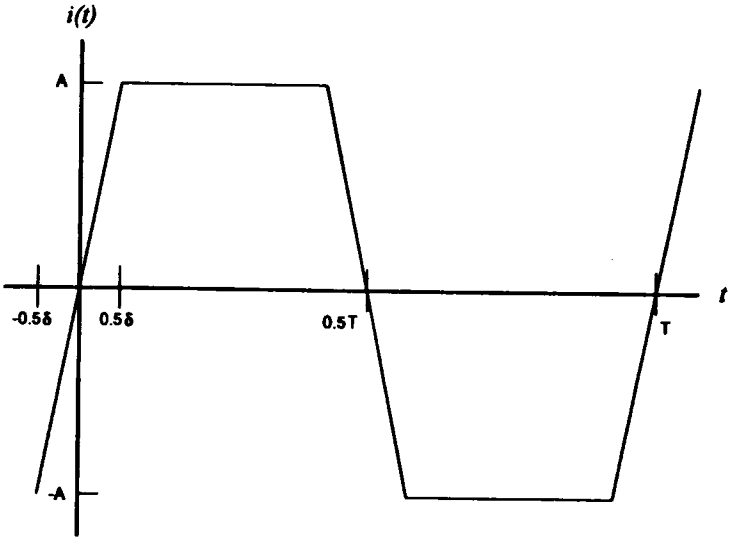

Figure 3. Trapezoidal Waveform.

The trapezoidal waveform in Figure 3 is convenient for use in testing K-factor calculation schemes since the K-factor of the waveform can be adjusted simply by changing the value of the transition time, δ. Applying equation (10) gives the ideal K-factor of the trapezoidal current waveform *i(t)*:

$$K = \frac{2T}{\pi^2\left(\delta - \dfrac{4\delta^2}{3T}\right)} \tag{14}$$

The trapezoidal current *i(t)* in Figure 3 has the following Fourier series up to the *Nth* harmonic:

$$i(t) = \frac{2AT}{\pi^2\delta}\sum_{h=1}^{N} B_h \sin\left(\frac{2\pi h}{T}t\right) \tag{15}$$

where B_h is defined as:

$$B_h = \frac{\left[\sin\left(\pi h \frac{T-\delta}{T}\right) + \sin\left(\pi h \frac{\delta}{T}\right) - \sin(\pi h)\right]}{h^2} \tag{16}$$

K_N is found by inserting equation (16) into equation (11):

$$K_N = \frac{\sum_{h=1}^{N} (hB_h)^2}{\sum_{h=1}^{N} B_h^2} \tag{17}$$

If the *Nth* harmonic is the highest harmonic that is to be included in a K-factor calculation, then a suitable input filter is a low-pass filter having a 4th-order Butterworth response with its 3dB cut-off frequency, f_C, slightly above the frequency Nf_1. The magnitude of the filter gain can be expressed as $G(f)$:

$$G(f) = \frac{1}{\sqrt{\left(\left(\frac{f}{f_c}\right)^4 - 3.414214\left(\frac{f}{f_c}\right)^2 + 1\right)^2 + \left(2.613126\frac{f}{f_c} - 2.613126\left(\frac{f}{f_c}\right)^3\right)^2}} \tag{18}$$

K_{Nf} can be approximated by computing equation (12) up to the *2Nth* harmonic:

$$K_{Nf} \approx \frac{\sum_{h=1}^{2N} (G(hf_1)hB_h)^2}{\sum_{h=1}^{2N} (G(hf_1)B_h)^2} \tag{19}$$

The block diagram of Figure 2 was implemented in PSpice using a fourth-order Butterworth low-pass filter. A trapezoidal waveform was used as the input. The results of the simulations, as well as calculations based on equations (14), (17), and (19) are shown in Table 1. In the table, T was normalized to have a unitless value of 1, and δ ranges from its maximum value of 0.5 (a triangle wave) down to 0.001 (a nearly-perfect square wave). N was selected to be 33 since this is a typical limit. It was empirically found that choosing f_C to be equal to $(N + 0.5)f_1$ gave good performance. The results of the simulations and calculations presented in Table 1 would be identical for trapezoidal waveforms scaled to 60 Hz.

$T=1$		$N=33$	$f_C=33.5$	
δ	K	K_N	K_{Nf}	K_{Nf}(SPICE)
0.5	1.216	1.201	1.201	1.201
0.05	4.342	4.105	4.073	4.079
0.01	20.54	12.39	12.28	12.43
0.001	202.9	13.93	14.07	14.33

Table 1. Results of K-factor Calculation Methods for a Trapezoidal Waveform.

5. FIR Filter Differentiators

The time derivative of a signal can be accurately computed with a finite-impulse-response (FIR) filter algorithm [9]. An ideal discrete-time differentiator FIR filter having $m+1$ samples has the following frequency response $G(e^{j\omega})$:

$$G(e^{j\omega}) = j\omega e^{-j\omega\frac{m}{2}}, \quad -\pi < \omega < \pi \tag{20}$$

The corresponding ideal impulse response, $g[n]$, is:

$$g[n] = \frac{\cos\pi\left(n-\frac{m}{2}\right)}{n-\frac{m}{2}} - \frac{\sin\pi\left(n-\frac{m}{2}\right)}{\pi\left(n-\frac{m}{2}\right)^2}, \quad -\infty < n < \infty \tag{21}$$

The ideal impulse response can be approximated by multiplying $g[n]$ term for term with a windowing function such as the Kaiser window, $w[n]$:

$$w[n] = \begin{cases} \dfrac{I_0\left[\beta\left(1-\left[\dfrac{n-\alpha}{\alpha}\right]^2\right)^{\frac{1}{2}}\right]}{I_0(\beta)}, & 0 \le n \le m \\ 0, & \text{otherwise} \end{cases} \tag{22}$$

where $\alpha = \frac{m}{2}$, and I_0 represents the zeroth-order modified Bessel function of the first kind.

The resulting $m+1$ filter samples $b[n]$ are therefore found by:

$$b[n] = g[n]w[n], \qquad 0 \le n \le m \tag{23}$$

It has been found that suitable values for m and β are 5 and 2.4, respectively. The corresponding values for a six-sample FIR filter are given in Table 2.

n	0	1	2	3	4	5
$b[n]$	0.0167	-0.1001	1.2277	-1.2277	0.1001	-0.0167

Table 2. FIR Sample Values

In order to produce a unity K-factor reading when the input signal is a pure sine wave at the nominal power line frequency, the values for $b[n]$ should be scaled as in equation (10) and divided by the sampling interval, T, so as to obtain the final filter sample values $c[n]$:

$$c[n] = \frac{b[n]}{2\pi f_1 T} \tag{24}$$

Odd values of m produce the best differentiator filters for a given number of samples. In addition, when m is odd, the differentiator output, $y[n]$, can be computed in terms of the differentiator input, $x[n]$, using the compact formula:

$$y[n] = \sum_{k=0}^{(m-1)/2} c[k]\left(x[n-k] - x[n-m+k]\right) \tag{25}$$

6. CONCLUSIONS

Bandlimited K-factor approximations can be calculated with adequate accuracy in the time-domain with analog or digital methods. These methods are simple enough that they should be easy to implement in low-cost hardware. Regardless of the method used to calculate K-factors, the number of harmonics that are included in the computations must be appropriately limited to avoid over specifying transformer requirements. K-factor computing instruments should allow users to select the number of harmonics to be included in a given calculation.

The K-factor by itself provides insufficient information for applying the methods outlined in the ANSI/IEEE C57.110-1986 standard. It would be very useful if transformer manufacturers could specify the percentage of the total losses in a transformer that are due to eddy currents when the transformer is loaded with the rated sinusoidal load current at the rated line frequency. Given the fact that winding eddy current losses have a transition frequency, current waveforms having identical K-factors could produce very different power losses in transformers that are constructed differently. Perhaps a standard method of identifying the transition frequency for eddy current losses could be developed so that this information could be published by transformer manufacturers as a guide to determining how many harmonics should be included in K-factor calculations for a given transformer.

There are some applications, such as electric transportation equipment, where the size and weight of transformers must be reduced as much as possible. In these situations, the AC loss and resistance formulas presented in [3], [5]-[8] can be used in combination with bandlimited time-domain K-factor calculations to optimize the designs.

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